Comparative analysis of the renewable energy generation forecasts in Poland and Portugal and their influence on the energy exchange prices

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Abstract

The share of the renewable energy and in particular the raising contribution of wind resources in the national energy-mixes of European countries showed a rapid growth over the last decade. Simultaneously, the beginning of the 20th century brought a revolution to the electricity trading systems, which from the state-owned structures transformed into free and competitive markets. Contemporarily, the privileged position of the wind energy producers is being weakened by their enforcement to participate in the market on equal terms, what entails the necessity of accurate production forecasting, what has been of secondary importance so far. The main aim of this study was to comparatively examine the wind generation forecasts in Poland and Portugal, as well as to verify their influence on the day-ahead market prices. The statistical analysis revealed significant deviations of the forecasted and actual wind production in both countries, which referred to the correspondent SPOT and balancing prices brought considerable financial losses from the perspective of energy suppliers. In the end, the influence of the wind generation forecasts on the SPOT prices has been examined by means of developed ARMA, ARMAX, NAR and NARX models. The results have shown that the usability of the information of forecasted wind generation is not unequivocal in developed SPOT prices models in Poland, mainly because of the randomness and volatility of recorded wind generation forecasts. However, in the case of Portugal, the forecasted wind generation occurred to be a valuable input in SPOT prices models, what has reflected in improved models accuracy.

Key words: Electricity Price Forecasting (EPF), wind power forecasts (WPF), SPOT market, ARMA, ANN

Resumo

A contribuição dos recursos eólicos no abastecimento elétrico dos países europeus tem revelado um rápido crescimento. Simultaneamente, o início do século XX trouxe uma revolução nos sistemas de energia elétrica, que passaram de estruturas monopolistas verticalmente integradas para mercados livres e competitivos. Atualmente, a posição privilegiada dos produtores de energia eólica está a chegar ao fim com a progressiva extinção das tarifas bonificadas, pelo que estes produtores estão a preparar a sua entrada no mercado elétrico. Esta nova realidade impõe a necessidade de uma previsão ainda mais precisa da produção eólica. O objetivo principal deste estudo é examinar comparativamente as previsões de geração de energia eólica na Polónia e em Portugal, bem como verificar a sua influência nos preços do mercado diário. A análise estatística efetuada revelou desvios significativos entre a produção eólica prevista e a realmente verificada. As penalizações valorizadas aos preços dos mercados diário e de desvios trouxeram perdas financeiras consideráveis. Finalmente, foi analisada a influência das previsões de geração eólica nos preços do mercado diário, por meio de modelos de previsão de preços (ARMA, ARMAX, NAR, NARX). Os resultados mostraram que a utilidade da informação da geração prevista de energia eólica não é inequívoca em modelos de previsão de preços do mercado diário desenvolvidos na Polónia, principalmente por causa da aleatoriedade e volatilidade das previsões registadas de geração eólica. No entanto, no caso de Portugal, a inclusão da geração de energia eólica prevista é uma entrada valiosa nos modelos de previsão de preços de mercado diário.

Palavras-chave: Previsão de Preços de Eletricidade, previsões de geração eólica, mercado SPOT, ARMA, ANN

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List of abbreviations:

ACF	Auto-Correlation Function
AIC	Akaike Information Criterion
ANN	Artificial Neural Network
AR	Auto-Regressive
ARIMA	Auto-Regressive Integrated Moving Average
ARMA	Auto-Regressive Moving Average
ARMAX	Auto-Regressive Moving Average with External input
BIC	Bayesian Information Criterion
BM	Balancing Market
CCP	Central Counter Party
CHP	Combined Heat and Power
DM	Daily Market
DSO	Distribution System Operator
EC	European Commission
ENTSOE	European Network of Transmission System Operators for Electricity
EPF	Electricity Price Forecasting
EU	European Union
IM	Intraday Market
LM	Levenberg-Marquardt
MAPE	Mean Absolute Percentage Error
MIBEL	Iberian Electricity Market
NAR	Non-linear Auto-Regressive
NARX	Non-linear Auto-Regressive with External input
OMIE	Spanish division of the Iberian Energy Market Operator
OMIP	Portuguese division of the Iberian Energy Market Operator
OSP	Operation Settlement Period
PACF	Partial Auto-Correlation Function
RES	Renewable Energy Sources
SARIMA	Seasonal Auto-Regressive Moving Average
TSO	Transmission System Operator
TGE	Polish Electricity Market Operator
WPF	Wind Power Forecasting

1. Introduction

1.1. General view on the topic

Sustainable Development of contemporary power system in the direction of Renewable Energy Sources requires facing the problems with unpredictability of their availability at a given time, what crossed with the time-variation of the load in the system and volatility of energy prices on the wholesale markets makes this issue strongly elaborate. One of the key aspects of reliable functioning of the power system is proper planning of the supply versus demand, which in light of the RES generation uncertainty becomes a struggle for responsible entities. Because of high unpredictability and instability, the inaccuracy of the e.g. wind generation forecasts entails consequences of two primary natures:

- Technical – Divergence of the forecasted planned RES generation impedes the TSOs to properly and accurately plan the fulfilment of actual demand by available power capacity.
- Economical – The uncertainty of the total forecasted wind power supply to the system translates into the uncertainty of the day-ahead and balancing market prices, what can result in financial losses of the market participants.

The study being the subject of this thesis will refer to the latter aspect. The fact that the wind energy influences the wholesale market is well known. This considerable impact is caused by relatively low marginal cost of the energy from windfarms, because in principle, the wind energy is not burdened with fuels costs. Therefore, an injection of the wind energy, featured by the lowest unit cost, shifts the supply-demand curve to the right, what results in reduction of the market prices, what has been illustrated in the Figure 1.

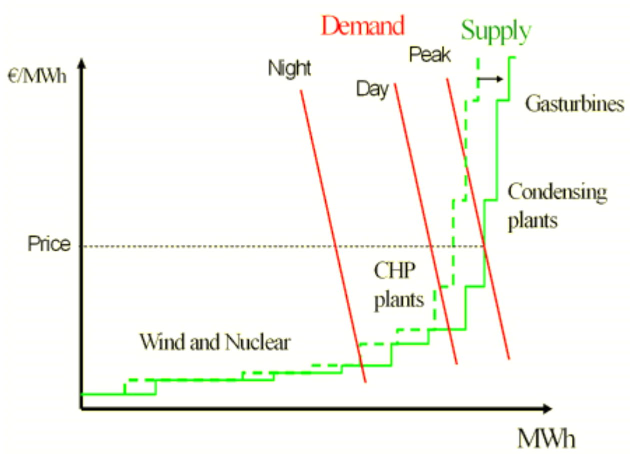


Figure 1 The influence of wind power on the SPOT market prices [1]

The green solid line represents the modified supply curve after injection of a given amount of wind power to the system, what is reflected in lower unit prices, compared to the dashed line (without wind power). As it can be observed in the Figure 1, the reduction of prices is of the highest level in the peak-time, while in the time of the demand-valleys this decrement is not so remarkable. In the situation of significant amount of wind power in the system, the conventional and CHP units, characterised by high fuel costs but being also the base load suppliers need to be manipulated.

1.1. Aims and objectives

In the literature review concerning this project, a wide range of studies concerning the predictions of wind energy as well as SPOT market prices have been found. Although usually, in the collected publications, a focus was put only on a single case of particular power system/power market. To the best knowledge of the author, a multi-subject studies have not been performed to reveal whether a given forecasting methodology applies to the separate cases of different national market environments, such as in Poland and Portugal. Despite the fact that the Polish and Portuguese wind power capacity in the system is similar in amount, when considering the share of this kind of energy the differences are substantial. Therefore, the thesis is aimed to address the following research questions:

- **What is the quality of overall wind power forecasts in Poland and Portugal and what are the most-distinguishing features of the forecasting errors?**
  Evaluation of the wind power prognoses will be made by means of statistical measures, which in quantitative way will give a picture on the accuracy, characteristics and tendencies observed. The comparison of obtained statistic description for both countries will allow to distinct the main differences between the forecasts and point out the main sources of uncertainty.
- **What is the cost of inaccurate wind power forecasting globally from the perspective of wind power producers in both countries?**
  Using the market and wind generation data available for both countries, a financial loss from inaccurate wind power predictions will be calculated, which will illustrate the range and importance of the appropriate and accurate wind generation forecasts.
- **To what extent the information of expected wind generation improves the accuracy of the wholesale market prices forecasts in both examined countries?**
  The verification of whether a wind generation has a value in predicting market SPOT prices requires a development of mathematical models. For the purpose of the study, the four models commonly used in dealing with market prices time series have been applied. The

modelling process will be carried out independently using data from Polish and Portuguese systems, what will allow to:

- Notice if the wind generation data is improving the SPOT prices predictions in both countries,
- Check if a given model can be successfully applied for the data coming from diametrically different power and market systems.

1.2. Document layout

In order to keep a comprehensiveness and readiness of the thesis, it has been written in a way allowing the reader to get the general context of the issues being examined, through the description of the methods used, ending on the results and conclusions:

Chapter 2: State-of the-art review on forecasting the wind power and SPOT market prices, as well as discussing the development of the wind energy in power systems in the light of national and European law and regulation. The chapter aims to reveal the main differences between Portuguese and Polish power systems, also taking into account the market environment and applied solutions.

Chapter 3 deals with the national wind generation forecasts published by the ENTSOE platform, which confronted with the actual measured data allowed to perform a statistical analysis of forecasting errors for both countries. The market data available on also on the ENTSOE platform combined with the calculated wind power forecasting errors made possible to obtain globally financial losses resulting from the inaccuracy of wind power predictions.

Chapter 4 has been devoted to the description of the SPOT market prices forecasting models developed in this project, with all the correspondent literature background, concepts and mathematic expressions.

Chapter 5 constitutes a collection of the obtained SPOT market hour-ahead forecasts results based on the models described in chapter 4, together with observations and comments for the period of December 2016 SPOT prices as evaluation dataset.

Chapter 6 includes the most important observations and key findings from the conducted study.

At the end of the document, in the appendixes, the Matlab codes of the SPOT prices forecasting models have been attached.

2. Renewable energy in the power systems

2.1. State-of-the-art review

This project concerns the two crucial problems in sustaining modern power systems, being a subject of intensive research within last decades, which are the Wind Power Forecasting (WPF) and Electricity Price Forecasting (EPF). The increasing share of wind energy in the power systems, together with tendency for equalization of wind power producers with all the remaining electricity market participants leads to the enhancement of interaction between the WPF and EPF. The impact of the wind power penetration into the power grid on the market player's behaviour and his decisions has been studied by Usaola (2008), who estimated that the uncertainty of wind power supplied to the system leads to significant financial outcome reduction of the energy producers, reaching 10% of maximum obtainable revenue [2]. The same finding has been obtained by Crespo-Vasquez et al. (2017), who in the evaluation of the model constructed for decision-making of the windfarm owner underlined the loss of the incomes resulting from the uncertainty of wind power output – in this case the losses were constituting 4% of the total earnings [3]. The mitigation of this negative impact resulting from unpredictability actual wind production has been addressed by González-Aparicio and Zucker (2015) by means of application of clustering and regressive techniques in order to narrow the wind power uncertainty ranges and optimize bidding strategies, what brought promising results on the example of a given windfarm in the Spanish market; in this case, the prediction error value was 2.5% [4]. The following publications indicate that the improvement of WPF becomes crucial from the perspective of enhancement of the effectiveness of market actions taken by its contributors.

2.1.1. Wind Power Forecasting

Among the studied publications, the authors distinguish two main focuses made for counteracting the instability and uncertainty of the wind power: (i) improving the models for wind power forecasting and (ii) setting up energy storage and reserve power units. Despite the remarkable wind power capacity in the system, the existence of conventional reserve power is necessary as the interventional tool for System Operators to sustain a balance between the demand and supply. The back-up of the conventional plants for production deteriorates the overall environmental impact of the wind energy installations [5].

The comprehensive and condensed study on WPF methods has been delivered by Xin (2011), what gives a view on numerous aspects related to this issue. Above all, the two main approaches in WPF models can be distinguished:

- Physical: the models are created based on the measurements, technical data of the windmills and air parameters provided by the weather prognoses.

- Statistical: including also the artificial intelligence, bases on the statistical models.

The importance of the WPF reveals in the variety of aspects, on which the proper wind power prediction has an influence. This variety entails the necessity of forecasting in different time horizons, depending on the target use [6]:

Table 1 WPF forecasting time horizons and corresponding applications [6]

Time Horizon	Target use:
Up to 30 min ahead	Electricity Market Clearing Wind Turbine control
Up to 72 hours ahead	Load Planning
Up to 1 week ahead	Unit Commitment Decisions
Up to 1 year ahead	Designing windfarms Maintenance schedules

Besides the abovementioned criteria, the wind power forecasts can be classified according to the area which is covered by the forecast (single windmill, windfarm, cluster of windfarms, geographical region) or character of model's input data (e.g. whether it is basing on a Numerical Weather Forecast or not) [6]. As the main opportunity of WPF improvement, Xin (2011) identifies a combination of the already developed, but yet separated statistical and physical models [6].

Another issue related with the WPF is its scalability, what has been pointed by Rasheed et. al. (2014). The author emphasizes that for two instances of exactly the same weather conditions in the windfarm the power output may differ significantly, what is caused by the complexity of the terrain on which the installation has been set up [7].

The fact that there are not universally optimal WPF models for all the windfarms has been highlighted by Banerjee et al. (2017), who demonstrated that the "best" modelling approach depends also on the evaluation criterion. Therefore, an optimization algorithms have been constructed in order to help the wind producer in finding the most appropriate modelling tool in each particular case [5].

The achievable decrement of the WPF errors entails the necessity of knowing the primary sources of uncertainty. In the paper published by Monforti et. al. the authors have undertaken the attempt to detect and weight the main factors affecting inaccurate wind power predictions. Besides the uncertainty of predicted weather conditions, the lack of comprehensive knowledge of the technical parameters of the wind farms and insufficient information on the wind fields are mentioned as the two main sources of WPF inaccurateness. Furthermore, it was found that the wind fields are of crucial importance when forecasting the wind power at national level [8].

2.1.2. Market Price Forecasting

The desire for knowing the market prices in advance with acceptable reliability has become naturally the subject of many studies. A remarkable focus has been put on forecasting the spot market prices [27], [29]-[30].

During the literature study for the purpose of this thesis, numerous approaches in forecasting the spot market prices have been found. Bessec et al. (2014) distinguishes three main modelling methods for predicting the energy prices on the market: equilibrium and game theory, simulation and time series forecasting methods [27]. Within these groups of methods, dozens of techniques have been developed, improved and merged with each other for obtaining the most reliable results.

The most widely used methods comprise the time series models, which, simply speaking, use the past observations of a variable to estimate its future values. The basic models for prediction of time series include smoothing methods like averaging and exponential smoothing (e.g. Brown's, Holt's Winter's methods). Although, the results of these models are burdened with relatively high prediction errors [27]. Throughout the numerous scientific works, the most attention is paid to the autoregressive (AR), Moving Average (MA) and their coupling - ARMA models, which successfully find an application in forecasting economy phenomena [24]. The ARMA model can be useful in stationary processes, which is characterised by a constant value of the average in the whole time domain. Since a lot of processes indicate a presence of a trend, the ARMA model can't be applied directly – this problem is solved by extension of the ARMA model by integration (I) term, which allows to transform the process with trend into a stationary one (ARIMA). Another difficulty comes from the seasonality of some time series in a short time, what has been addressed by Seasonal (SARIMA) model.

The aforementioned models work on a basis of a significant assumption, that the observed variable is of a constant volatility, it is, its variance mean amplitude doesn't change in time (homoscedasticity). However, the amplitudes of variance may be influenced by some external and temporal factors. This obstacle is counteracted by applying the GARCH model, which improves the predicting process of time series with unexpected spikes [11].

In the age of computers, the Artificial Neural Networks (ANN) as a branch of Computational Intelligence (CI) become increasingly popular in all range of predictions, mainly because of their forecasting accuracy and availability of dedicated software. Neural networks can be used successfully in circumstances which prevent the statistical methods to be applied. As one of the main advantages of the ANN one may mention its adaptability for complex, dynamic and nonlinear relations. On the other hand, Weron (2014) emphasises that the neural networks can be susceptible for unexpected, rapid changes in the process. The wide range of available ANNs usually is classified due to the architecture or learning algorithm [33]. Besides that, Weron (2009) points out the hardness of comparison of forecasting efficiency of different CI methods among each other, because of the calibration conditions. Kolmek and Navruz (2012) performed the parallel forecasting processes for the Turkish energy spot prices by means of the ARIMA and ANN model with the learning dataset constituting historical observations for time-range of 342 days. The latter model was burdened with smaller error, what was concluded as a main predominance of this method with respect to statistical techniques (15.60% MAPE for ARIMA vs 14.15% MAPE for ANN, calculated for a time horizon of 1 week) [33].

Another approach finding an application in spot prices modelling is a Wavelet decomposition. The time series is decomposed into components of different frequency. Those components may have a different importance in providing the information to the model, what entails the necessity of assigning the proper weights to particular waves of given frequency in order to reflect possibly the most accurate projection explained variable. The low frequency components represent the global information (e.g. trends), whereas the high frequency waves constitute an input of the detailed information [35].

In his explicit study, Weron (2014) lists the strengths and weaknesses of the electricity spot prices in terms of the aforementioned methods, often being individual for a particular modelling approach. The compensation of the weaknesses may be achieved by combining the modelling tools by means of dedicated computer software. The hybrid EPF models become ascendingly applied in forecasting problems, often resulting in performance improvement [36]. As the examples of hybrid methods of spot price forecasting, the studies of Zhongfu Tan et al. (2010, wavelet transform combined with ARMA and GARCH models) and M.Shafie-khah (2011, wavelets, ARIMA, NN combination) can be pointed, resulting in promising outcomes of the proposed junctions [36].

The comprehensive literature study of Weron (2014) lists the areas of EPF, which still have a potential of improvement, as well as some overlooked issues which restrain achievement of better forecasting results, which are inter alia:

- optimization of input variable set from the wind range of possibly influential factors
- the approach to reflect the seasonality and periodicity in different time horizons doesn't keep up with the development of other EPF factors being more deeply analysed
- prediction of atypical, irregular events like price spikes, having the reserve margin as an explaining variable
- directing of the study focus from the point forecasts to the interval and density forecasts
- extension of the forecasting horizon with subsequent improvement of long-range uncertainty and risk

- threshold forecasting, which instead of predicting exact, definite values of energy prices, estimates the thresholds of the price for planning purposes
- combination of forecasts (forecasts pooling), which indicates promising results, but has not been scientifically supported yet to an adequate level
- finally, the necessity of establishing the unequivocal ground for testing and evaluating of different forecasts. The author points the lack of consequence in model building process and diversity of evaluation methods, what disables the direct comparison and efficiency measure of considered forecasting models.

Many researchers put the question of the influence of wind generation forecasts on the wholesale market prices and built dedicated for it forecasting models [37]. Nevertheless, to the best knowledge of the author, there have not been made a one integrated study comparing directly the economically and technically different power systems. Poland and Portugal developed diametrically opposed energy mixes, and the energy prices may be driven by various factors of different importance. The impact of wind energy generation on the market prices is a mutual problem for both countries; the question which has to be answered is to what extent the wind power injected to the system affects day-ahead prices.

2.2. Portuguese and Polish RES sector in reference to the European Union

Within recent several decades, the European energy systems have undergone ground-breaking changes, mainly because of the outspread environmental consciousness of its citizens, as well as of representing them politicians. Following The European Commission's directive [14], called ordinarily "3x20", (imposing the reduction of the CO_2 emission by 20% with simultaneous increase of the renewable energy share in the energy mix and the widely interpreted energy efficiency also by one fifth by the year 2020), recently being updated for the 2030 horizon, the European Union countries have to face many hindrances on the way to achieve the established commitment. The accomplishment of the aforementioned tasks requires enormous amount of investment not only in the infrastructure, but also in the management and administration, what influences the whole economy [1].

According to Eurostat (2016), the majority of the countries are on a good way to achieve the aimed thresholds in the year 2020. Moreover, some of them have already met the EC directive's requirements, continuing to develop the improvement of the national energy sector. In Figure 2, the percentage share of the RES for Portugal, Poland and entire EU has been shown [15].

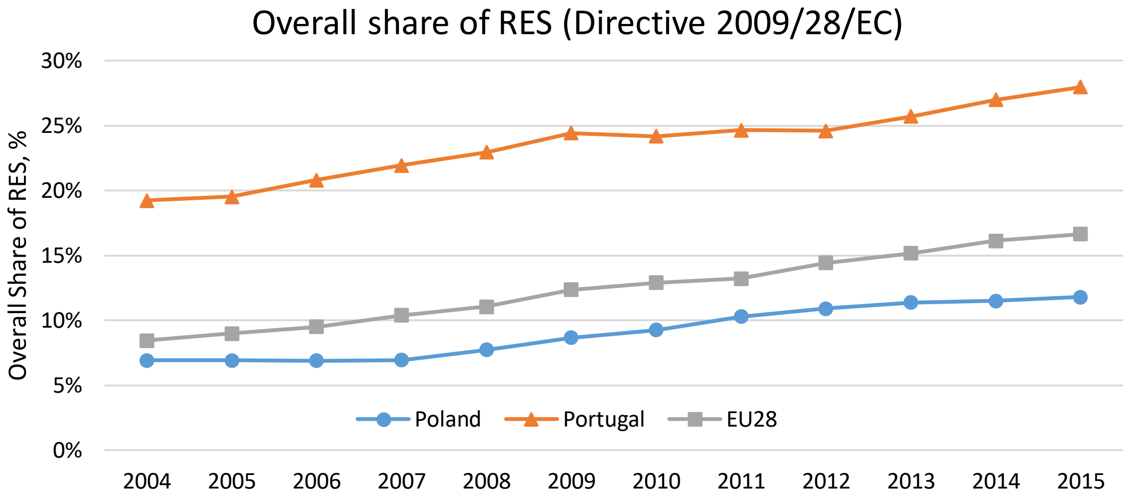


Figure 2 Overall share of RES in Portugal, Poland and EU28 in the years 2004-2015 [15]

Portugal is a country characterized by very high share of the RES in the total energy mix, reaching 28% in 2015, while Poland represents steady growth in this area as well, being slightly below the EU28 average (11,8%). In this point it has to be stressed that direct comparison on the RES share is improper, since throughout the continent, the countries have developed diverse and specific energy systems, depending mainly on available domestic resources. Another key factors determining the shape of energy systems are climate conditions (e.g. temperature), as well as the maximum energy independency. Despite the fact that in Poland considerable increment of RES in total energy production is observed, the dominating primary energy source constitutes coal, which is the most CO_2-intensive energy production technology. Contrarily to Poland, Portugal's energy mix is diverse, with no single dominant energy production technology [17],[18].

The continuation of the EU plans for clean and sustainable energy technologies requires the adequate and optimal utilization of the wide range RES technologies in order to make it not only efficient, but also profitable. Poland and Portugal are countries very distant geographically, what results in different climate conditions and in different available ways of renewable energy generation. Taking into account the average annual wind speed, in both countries the obtained measures are similar (0-4 m/s) [16]. On the other hand, the solar energy in Portugal has much higher potential for utilization, what measures the solar irradiation (1800 kWh/m^2 in Portugal, around 1200 kWh/m^2 in Poland). The pie charts generated on the basis of the Eurostat reports show the decomposition of the electric energy generation from RES with regard to particular technologies [15].

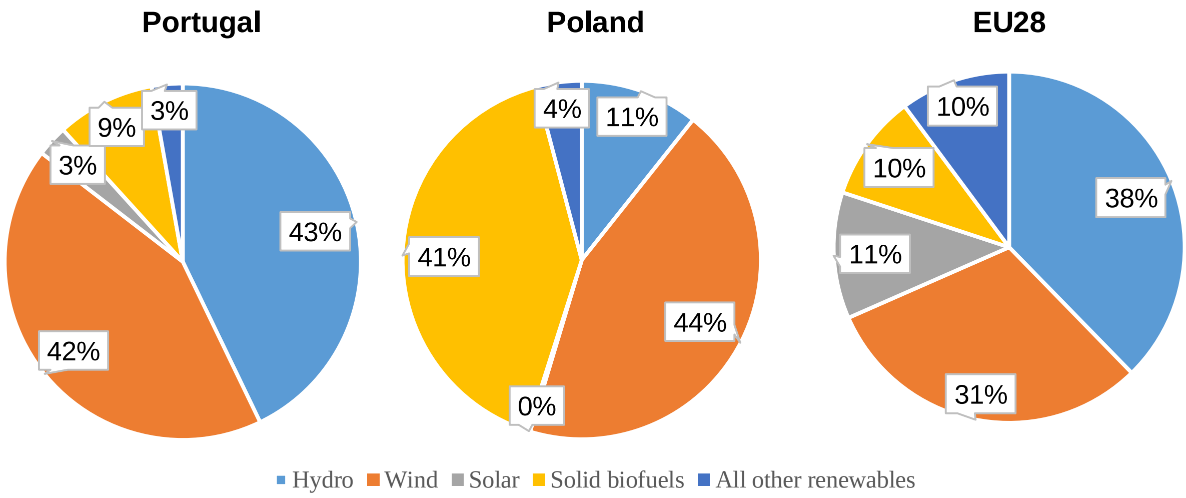


Figure 3 Renewable electric energy production per type in 2015 [15]

In the year 2015, the Portuguese electric energy generation branch was injected by 2400 ktoe of energy from RES, dominated mainly by wind and hydro power. On the other hand, the Polish renewable electric energy was produced mainly from wind and solid biofuels in the overall of 1890 ktoe. As it was explained in IEA report [18], the relatively high share of the biomass may be resulting from the possibility of burning or co-burning biomass in the conventional coal-fired boilers, what still is considered as the renewable energy technology. A significant part in the RES, taking into account the entire EU, constitutes the solar energy, equal to 11% of total in 2015. The share of solar energy in the Portuguese RES structure is of a relatively small importance, even though this country has one of the biggest potentials of utilization of energy from the sun [17].

Both Portugal and Poland, as well as the entire group of 28 EU show a considerable share of the wind energy in the total RES electricity production. This kind of energy generation became commonly chosen technology in the energy systems, due to its relative simplicity, time of construction and accessibility of wind resources.

2.3. The influence of wind generation on the energy markets

Wind power is one of the most commonly applied technologies in renewable energy generation. The increasing share of the wind energy in the overall production amplifies also the difficulty of balancing the system, mainly because the accessible wind resource is unstable and troublesome to predict in longer time. This entanglement is enhanced by the increasing production capacity of windfarms. In the Figure 4, the actual power capacities of wind energy producers can be read, reaching over 5GW both in Poland and Portugal in 2016.

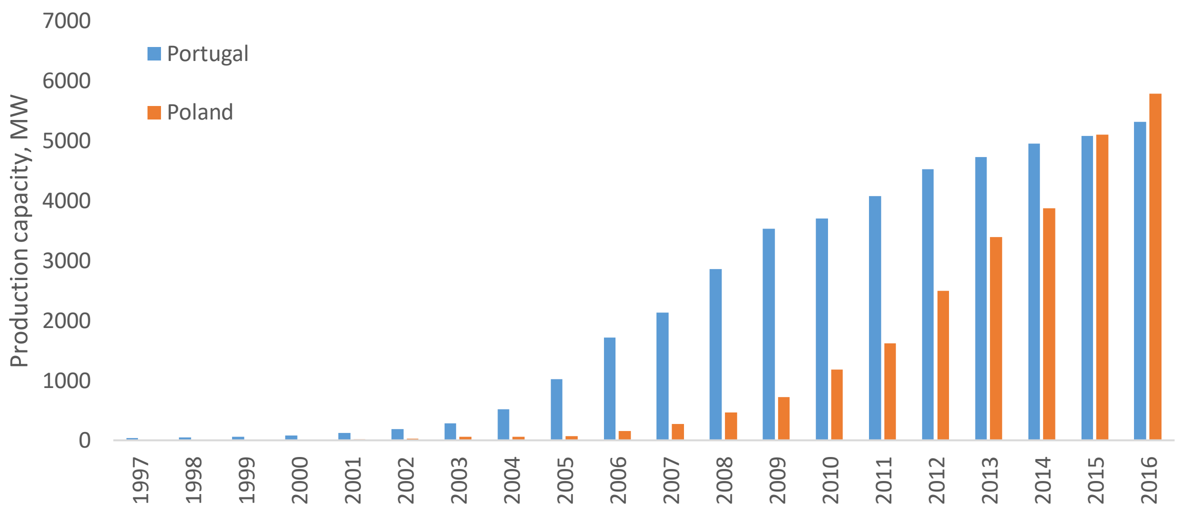


Figure 4 Wind production capacity in Poland and Portugal in the time range of 1997-2016 [19]

When comparing the dynamics of wind energy development in both countries it can be noticed that Polish wind energy sector is undergoing rapid expansion, while the Portuguese one is increasing moderately, resembling proportionality with respect to time.

Contemporarily, when the electricity is traded on the wholesale markets, the uncertainty regarding the amount of energy supplied to the system influences directly the prices on the market. Every excess or deficiency of energy in the system has a consequence in the financial outcomes of energy suppliers/receivers [2].

In order to avoid the financial losses, all the wind energy related subjects in the system are continuously working on the tools allowing to forecast the wind generation as well as possible. The emphasis on the wind generation predictions is made both in microscale (particular wind farms) and macroscale (the overall energy supplied to the system). The volatility of wind energy generation comes mainly from the dynamic weather conditions, which also are difficult to be accurately foreseen. Popławski (2014) highlighted that the development of wind energy technologies goes parallel with the development of wind power forecasting tools, especially in the Western Europe institutions. The constantly improving prognoses are mainly in the interest of energy Systems Operators (SO), who release the forecasts results publicly. Nevertheless, the methodology used is often restricted or covered by the patent laws.

It also should be pointed, that the uncertainty of the energy produced in a system has not only negative consequences. The presence of the day-ahead market, as well as the balancing market may be a field to multiply the member's revenues, who may speculate the prices and gain profits only from buying/selling the energy depending on the market or imbalance prices [2].

Usaola (2008) describes the sequential pattern of the arrangement of the majority of electricity markets, depending on the bidding horizon. The earliest bids take place from months to days before the Operational Settlement Period (OSP) on the Forward Market, further, the Daily Market commitments are taking place up to several tens of hours before the OSP. Finally, the shortest lag between the commitment and the OSP (hours) is characterizing the Intraday Market (IM), as well as corresponding Balancing Market (BM). The wind generation forecasts perform acceptable and reliable result mostly up to one day ahead, thus, the wind generation will affect mostly the Daily and Intraday Market [2].

2.4. Contemporary Structure of European Energy markets

Electric systems origins took place about one hundred years ago, giving an incipience and basis for constantly undergoing development of civilization. The most rapid development of electric systems has been observed just after the Second World War, when centralized, state-owned structures have been created in order to completely control all the electric energy subsystems, from generation to the final delivery. It was motivated mainly by the strategic importance of the electricity as a good, especially from the perspective of public utilities and the significance of energy supply during war. The first country which desisted the monopolistic, state-owned electric system and turned into direction of competitive markets was Great Britain, by the establishment of the Electricity Act in 1989. This unprecedented change of approach became a trigger for ground-breaking changes in other European which contribute to the actual shape of electric energy systems [1].

One of the main disadvantages of the centralized, state-owned system is its susceptibility for the political manipulations, what could be observed, for example in the Soviet Union countries in the end of XX century. Because of the centralized management of all the parts of the system, its priorities could be affected or re-defined by the current authority. Moreover, the imbalance between the technical and economic effectiveness of the system towards the former one. The competitive market allows to avoid the possibility of this imbalance to occur [1].

Contrary to the abandoned monopolistic approach, contemporarily, the electric energy is considered as a commodity rather than a good. Furthermore, the distinction between electricity as a product and electricity delivery as a service is fundamental of the competitive market existence [1]. Despite the fact that the energy industry reforming may take different forms with relation to a particular country, Mielczarski (2000) highlights the universal aims of energy systems restructuring:

- Overall decrease of the energy prices
- Improvement of the economic efficiency of the energy systems
- Enhancing the innovative solutions in energy systems

- Ensuring the possibility of choosing the electricity provider
- Establishment of the customer's legal protection
- Increase in the quality and reliability of electricity supply

All the mentioned aims are possible to achieve in the competitive market in the conditions of equal treatment of its participants, unrestricted market access and unbidden prices derived from the demand-supply balance [1].

The concept of the liberated, deregulated energy market has been adopted in all the EU countries within recent 20 years, although the process of restructuring could take different forms and have different motivation [10].

The approach of competitive wholesale market has evolved to the international scale. Kopsakangas-Savolainen et. al. (2012) describes the example of Nordic Power Market (Nord Pool), being a consolidated energy market for 5 Northern Europe countries (Sweden, Denmark, Finland, Norway, Estonia) since 1999. This unification made the Nord Pool market the World's largest (considering volume). The Nord Pool market constitute about 300 participants, who are not obliged to buy/sell on the market, although, the major part of transactions (70%) has been made through the spot market. Because of the common management of the interconnections between the countries, as well as the possibility to trade the energy between the subjects from other countries, the overall efficiency of the system has improved [10].

2.5. Characteristics of Polish and Portuguese energy exchange institutions

Day-ahead market type is commonly used in commodity, currency or shares market – the transaction is made up to two days before the physical execution. What makes the energy day ahead markets unique is the inability of the storage of the commodity. This entails the necessity of continuous balancing the supply and demand [11].

In order to ensure the wide access to the market information, to maintain the clearance and transparency of the transaction rules, the state-controlled institutions have been established. The existence of these companies are a foundation for the fair and equitable energy exchange systems. Below, the short characteristics of Polish and Portuguese energy exchange institutions have been shortly described.

The Towarowa Giełda Energii (TGE) has been stablished in 1999 as a response of the Polish government for the necessity of institution managing the energy trading on emerging liberated, competitive market. Half a year after commission, the spot market has been launched for market participants. Currently, TGE is the only company licensed by the state for managing the energy exchange. During two recent decades, TGE expanded their area of activity by the emissions market exchange and origin certificates trading platform. In 2008 TGE introduced the Commodity Forward Instruments market, which allows to maintain relatively constant prices in a wider time-horizon, as well as to optimize the costs of sales/purchase of the energy [12].

While the TGE activities are limited to the country borders, Iberian Electricity Market (MIBEL) is an example of the energy market expanded to the regional scale, being a result of joined efforts of Spanish and Portuguese governments. This convergence, having origins in 2001, allows every market participant to make deals with subjects from all over the Iberian Peninsula. The integration of hitherto separated energy systems required a series of undertakings, including the harmonization of the electric network, law and economic environment. In 2004, the both parts of the Santiago Agreement declared the creation of two sub-institutions, responsible for different aspects of proper functioning of the system as a whole. These were the OMI-Polo Português (OMIP), responsible for the forward market and OMI-Polo Español (OMIE), which was brought to existence to manage the spot market. The timespan between the Santiago Agreement and the launch of the Iberian Market (1 July 2007) was influenced by many factors, including political changes. Creation of the new companies enforced the involvement of already exiting, yet separated market operators (OMEL - Operador del Mercado Ibérico de Energía, Polo Español and OMIP–Operador do Mercado Ibérico SGPS) to transfer some of the business branches to the newly founded enterprises. As a completion to the unified market, the OMIClear corporate has been established. The OMIClear acts as a Central Counterparty (CCP) to all the operations taking place on the market, guarantying transparency and proper risk management. In other words, OMIClear is an interconnector between buyer and seller, assuring that all the party’s commitments are going to be fulfilled. All the mentioned Iberian Electricity Market companies are not independent itself; their shares are distributed between the remaining companies. The graphical representation of the MIBEL has been shown in the Figure 5 [13].

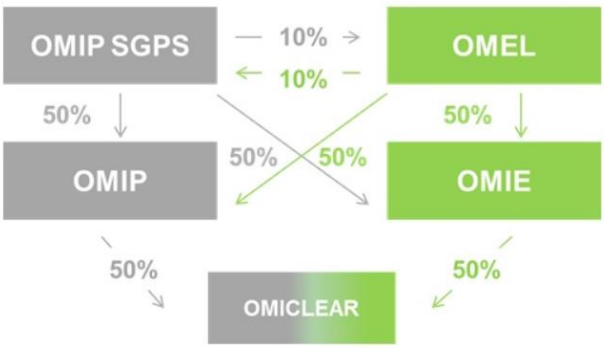


Figure 5 The organization structure of the Iberian Electricity Market with shares distribution

3. Polish and Portuguese wind power forecasts analysis

3.1. Verification of the Polish and Portuguese wind power forecasts by means of statistical measures

Numerous publications concerning the wind power forecasting underline the significant contribution of windfarms in the energy systems and associated difficulties [20],[21]. As the main cause of those difficulties, the instability of the energy production is considered. Dependency on weather conditions, and thus, on weather forecasts amplifies complicity of effective projection of wind generation. The System Operator requires in advance the planned generation of windmills in order to balance the demand and supply of electric energy, optimize the loads and plan the power reserves. For this reason, the wind energy producers are obliged to provide the System Operator with the plan of production for a given time ahead. Accurate forecasts, both in micro-and-macro scale, aimed at the minimum prediction error are necessary to maintain a reliable electric energy system [22].

3.1.1. Statistic description

In this study, the statistical analysis of the aggregated wind generation forecasts for Poland and Portugal has been conducted. The day-ahead forecasts together with actual generation of wind in the energy systems of both countries have been collected. The figures for analysis were sourced from the European Network of Transmission System Operators for Electricity (ENTSOE) website, which provides a wide range of up-to-date, energy systems and markets data. The data set used for statistical description was of the 1-hour fragmentation, encompassing the entire calendar year 2016, what gave 8784 records in total for each variable. The prediction error is expressed as the difference between the realization of the variable and its corresponding forecasted value for a given instant *t*:

$$e_t = P_t - P_t^* \tag{1}$$

Where e_t is the absolute error of the forecast and P_t, P_t^* represent the actual wind power in the system and its day-ahead prediction, respectively, expressed in MW. The error value has been calculated for each hour of the 2016 year and divided into ranges to present the error distribution on a histogram. The analysed forecasts has been published by the ENTO-E platform, which collects the Power System data from all the associated TSOs.

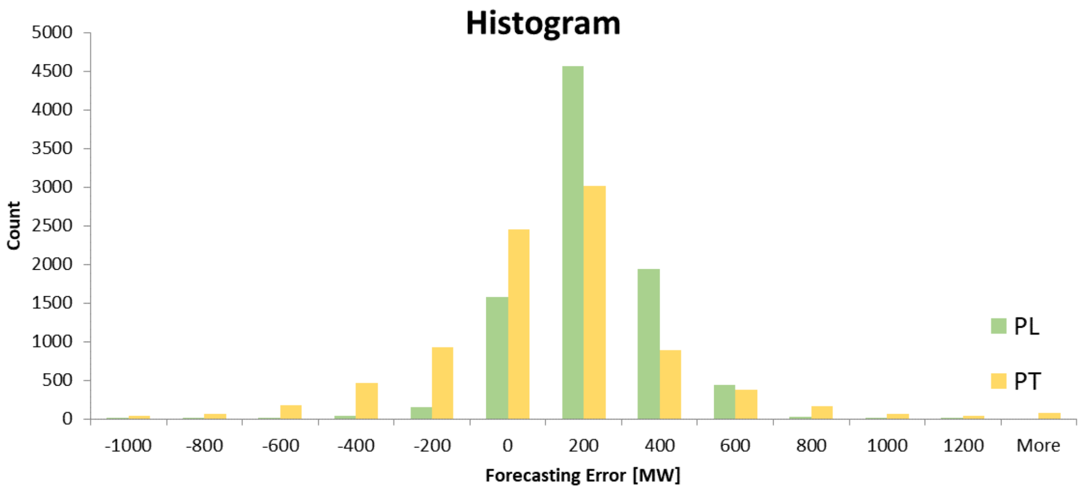


Figure 6 Aggregated wind generation error forecast distribution for Poland and Portugal, 2016

The main conclusion from visual analysis of the above chart is that the wind generation forecasts are generally underestimated in both countries, since the result of subtraction of the forecasted from actual value of power takes positive value, between 0 and 200 MW. To get more deep view into the characteristics of the error distribution, the statistical parameters of the examined dataset have been determined. Besides the minimum and maximum values in each of the samples, the average value of the forecasting error has been calculated, according to formulae:

$$\bar{e} = \frac{1}{N}\sum_{t=1}^{N} e \tag{2}$$

Where $\bar{e}$ represents the average error and N expresses the total number of observations taken into account. The next determined parameter is median, which is the value in an ordered set of values (here forecast errors), for which there is the same number of observations with higher and lower values. The statistic description includes also the dominant, which is the most frequently occurring value in the set of observations. Standard deviation σ of the forecast errors is calculated as follows. It describes the volatility of the examined sample:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(e_t - \bar{e_t})^2} \tag{3}$$

N stands for the total number of observations, which in this study was equal to 8784. Standard deviation has the same unit as the unit of the observed quantities. The concentration of the values around the average value is defined by the kurtosis K (4), while the asymmetry of the distribution graph is assessed by the Skewness S (5):

$$K = \frac{\frac{1}{N}\sum_{t=1}^{N}(e_t - \bar{e})^4}{\sigma^4} - 3 \tag{4}$$

$$S = \frac{N\sum_{t=1}^{N}(e_t - e)^3}{(N-1)(N-2)\sigma^3} \tag{5}$$

One of the most common ways to evaluate the accuracy of the forecast is the Mean Absolute Percentage Error (MAPE), which measures the relative inaccuracy of prediction:

$$MAPE = \frac{1}{N}\sum_{t=1}^{N}|\frac{e_t}{P_t}| \cdot 100\% \tag{6}$$

The statistic description results have been collected in the Table 2, both for forecasting errors in Polish and Portuguese wind generation.

Table 2 Statistical description of the wind Energy forecast error for Poland and Portugal in 2016

Statistical Measure	Poland	Portugal
Average, MW	118.23	7.52
Median, MW	98.00	8.00
Dominant, MW	71.00	3.00
Standard deviation, MW	173.00	353.30
Kurtosis	7.17	13.02
Skewness	0.399	-0.307
Amplitude, MW	2978.00	5593.00
Minimum, MW	-1189.00	-3659.00
Maximum, MW	1789.00	1934.00
MAPE, %	15.00	24.20

Analysing the above table one may conclude that both Polish and Portuguese forecasts are underestimated, because the average values exceed 0 significantly, especially in Poland, reaching 118MW. Despite the fact that the average, median and dominant are much higher in Poland with relation to Portugal, the Portuguese forecast is characterised by higher volatility, what indicate the values of kurtosis and standard deviation. Comparing the kurtosis of both forecasts errors, the higher tendency for clustering of the error close to the average value occurs in the forecast prepared for Poland. In both cases, the extreme values of the forecast error reached gigawatts. Comparing the MAPE values, the Polish forecasts perform considerably better, reaching 15% on average. The skewness values of examined samples took opposite sign for Poland and Portugal, what means that the asymmetry of the error distributions has opposite direction. Comparing the wind generation forecasts error (up to 2GW) with the overall generation rates in the countries (reaching up to about 10GW in Portugal and 50GW in Poland) leads to the question in what extend these forecast influence the total energy system and energy markets.

3.1.2. Theil divergence of the wind generation forecast [24]

Calculation of the Theil coefficient allows to have an insight in the particular sources of the total forecasting error. The general formulae for determining the Theil I^2 value is described below:

$$I^2 = \frac{\sum_{t=1}^{N}(P_t - P_t^*)^2}{\sum_{t=1}^{N} P_t^2} \quad (7)$$

The square from the Theil coefficient gives an information about average, relative forecast fit. The overall value of I^2 can be decomposed in three parts, representing different characteristic of the approximation error:

$$I^2 = I_1^2 + I_2^2 + I_3^2 \quad (8)$$

Where the individual components can be determined following the formulas 9, 10 and 11:

$$I_1^2 = \frac{N(\bar{P}_t - \bar{P}_t^*)^2}{\sum_{t=1}^{N} P_t^2} \quad (9)$$

$$I_2^2 = \frac{N(\sigma - \sigma^*)^2}{\sum_{t=1}^{N} P_t^2} \quad (10)$$

$$I_3^2 = \frac{2m\sigma\sigma^*(1 - R)}{\sum_{t=1}^{N} P_t^2} \quad (11)$$

where *R* represents the Pearson's coefficient of linear correlation between the time series P_t and P_t^*. The I_1^2 component measures the influence of the systematic error (improper adjustment of the average), I_2^2 represents the impact of the improper elasticity of the forecasting model (the estimated variance of the explained variable was inappropriately mapped), while I_3^2 reflects the fallibility of the model in spotting the points, where the trend of variable changes. Since the three parts of Theil divergence divided by I^2 sum up to 1, the percentage shares of the total value represent the relative influence of particular error root in general. The results of calculations have been gathered in Table 3.

Table 3 Theil divergence calculation results for Polish and Portuguese wind generation forecasts

parameter	PL	PT	parameter	PL	PT
I^2	0.01465	0.04176	I_1^2/I^2	31.84%	0.05%
I_1^2	0.00466	0.00002	I_2^2/I^2	3.84%	0.08%
I_2^2	0.00056	0.00004	I_3^2/I^2	64.32%	99.87%
I_3^2	0.00942	0.04171	$\sum$	100%	100%

Analysing the above table, the main cause of the wind generation forecast uncertainty is the directional divergence, both for Portugal and Poland, contributing in the total forecast error with shares of 64.32%

and 99.87% shares, respectively. In other words, the wind forecasts perform wrong when, for example, the generation suddenly starts to decrease after some period of increment. Additionally, a considerable part of the error in polish forecast (31.84%) is caused by the misestimating of the average value.

3.2. Determination of financial losses resulting from wind power forecast deviations

The rapid development of the wind energy made it one of the most commonly used RES technology worldwide, what was influenced mainly by extraordinary conditions of participation in the energy systems. As the most considerable factor for the wind energy expansion one may consider the politics. Besides the „3x20” climate pack, described in section 1.2, the EU imposed the mandatory purchases of energy from the owners of the windfarms (2001/77/EC directive). Even though the wind energy sellers are obliged to make a day-ahead production plans [25], the certainty of selling the entire produced energy make the accuracy of predicted generation unessential. The main sufferer of the significant deviations between the wind generation plan and its realization was the DSO, who in the case of energy imbalance in the system had to find alternatives for mitigate the inadvisable variations. The harmfulness of wind generation uncertainty can be noticeable especially in the countries, where the windmills take significant part in the total energy mix. The blackout phenomenon may be a consequence of extremely imbalanced energy system, what is given by the example of Germany [22].

Originally, in accordance with the 2001/77/EC directive the governments have implemented individual energy pricing mechanisms as incentives for wind energy investors. In Portugal, the feed-in tariff has been introduced, which regarded the windfarms not older than 15 years which haven’t produced more than 30GWh from the moment of its commission [26]. On the other hand, in Poland, the Energy Regulation Office has set the arbitrarily set uniform energy price, which in the worst case of high imbalance was differing from the contracted price insensibly [25]. All in all, the energy prices in Poland and Portugal were independent from the day-ahead market.

Nowadays, the aforementioned approach of wind energy pricing has changed into the trend for considering the wind energy producers as regular participants of the wholesale markets, as well as intra-day and balancing markets. This alteration raises the wind generation forecasts to the key-importance level. As the maximization of the economic income is a priority for the wind energy producers, the appropriate production prognoses are the foundation for effective operations on the market [26].

In the most of the cases, the wind generation forecasts are characterized by the short-term time horizon, up to 24hours ahead. The hour-ahead forecast, together with the intraday market platform may be a tool for modifying the day-ahead plan and its execution, nevertheless the participation in the intraday market brings the risk of reaching the limit values. The intraday market liquidity and reliability of the day-ahead forecast became two contrary decision variables to balance [26].

Having the total wind generation day-ahead forecast and actual values of wind energy injected to the system (ENTSOE) for hourly fragmentation in 2016 calendar year, the total financial losses resulting from wind energy day-ahead plan deviation are going to be determined, both for Poland and Portugal. For the purpose of this study, it was assumed that the wind energy sellers do not have any forward contracts. Additionally, the wind producers rely entirely on the day-ahead plan and do not participate in the intraday market.

At this point it has to be underlined that there are two divergent approaches of the imbalance pricing in Poland and Portugal. In Poland, there exists only one, uniform imbalance market price. Contrarily, in Portugal, two imbalance prices can be distinguished: the lower and upper imbalance price, depending on the imbalance character (deficiency or surplus).

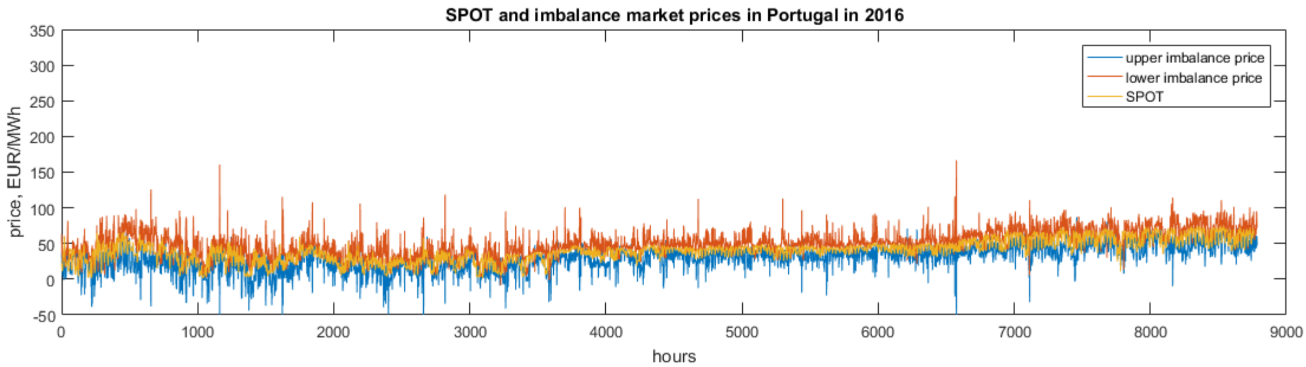


Figure 8 SPOT and balancing market prices in Portugal in 2016

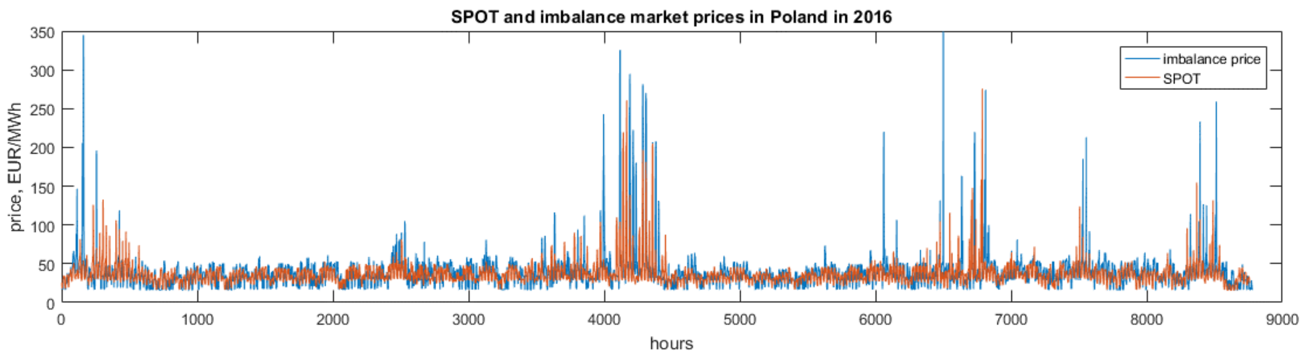


Figure 7 SPOT and balancing market prices in Poland in 2016

Both Figure 7 and Figure 8 reveal distinctive features – in the case of Poland, several examples of drastic peaks of the prices can be observed, reaching around 350 EUR/MWh. In the Portuguese market, the extremum reaches up to 175 EUR/MWh. Furthermore, what was often observed in the case of Portugal and did not take place in Poland is the negative price of energy.

Following the mentioned dissimilarities, the calculation of the financial loss has to be defined separately for both countries. the total loss of production plan deviation for Poland will be estimated as follows:

$$L = \sum_{t=1}^{N} |E_t - E_t^*| * (I_t - M_t), \qquad for\ E_t - E_t^* < 0 \tag{12}$$

$$L = \sum_{t=1}^{N} |E_t - E_t^*| * (M_t - I_t), \qquad for\ E_t - E_t^* > 0 \tag{13}$$

Where: L – financial loss, E_t, E_t^* - actual and planned wind energy, M_t – spot market price, I_t – imbalance price.

The imbalance price in Portugal will depend on whether there is a surplus or deficiency of the produced energy with respect to the day-ahead plan:

$$L = \sum_{t=1}^{N} |E_t - E_t^*| * (I_t^U - M_t), \qquad for\ E_t - E_t^* < 0 \tag{14}$$

$$L = \sum_{t=1}^{N} |E_t - E_t^*| * (M_t - I_t^L), \qquad for\ E_t - E_t^* > 0 \tag{15}$$

Where I_t^U and I_t^L are upper and lower imbalance price, respectively.

The aggregated financial losses have been calculated for the entire calendar year 2016. Moreover, the number of hours during which the wind generation forecast deviation brought benefits (the financial loss L took negative values) have been counted. The calculation results has been collected in Table 4:

Table 4 Financial losses resulting from wind generation forecast deviation in calendar year 2016

	PL	PT
Total financial loss, €	1 900 592	20 812 563
Number of hours, when forecast deviation brought benefits (L<0)	4510	814
Total volume of imbalanced energy, MWh	1 358 182	1 944 235
Unit cost of imbalanced energy, €/MWh	1.40	10.70

Although the total wind generated imbalanced energy volume in 2016 is not significantly different between Poland and Portugal, the financial losses in the case of the Iberian country reached almost 21 million €, what was over ten times more than in the case of Poland. Despite the fact that the yearly-aggregated forecast deviation brought losses, there were hours, when it resulted in positive income (L<0). In Poland, more than a half of hours in the year actually brought benefits from wrong planning (4510 times in 8784 of total). To directly compare the results, the unit costs of the forecast error has been calculated by division of the total imbalance by the total financial loss in 2016. As it is shown in the Table 3, the 1MWh planning error in Portugal resulted in 10.7€ of cost, what was almost 10 times in comparison with Poland. Such a significant discrepancy comes probably from the imbalance pricing system, which is different in these countries. In the circumstance of single, uniform imbalance price, there exist a considerable probability of gaining incomes from selling energy on the balancing market instead of the spot market. The Polish wind energy seller may speculate about the difference between the spot and imbalance prices, what may encourage him to intentionally distort the production plan. Contrarily, the Portuguese energy seller may expect only the negative outcomes from inappropriate production plan.

4. Spot market prices forecasting models

The necessity of accurate wholesale market prices forecasting is supported by the importance of this kind of information to all the electricity market stakeholders. Reliable predictions help the market participants to plan the activities and resources in different time horizons. Moreover, a price forecasting may be a basis for regulatory constraints and limits imposed by the relevant market bodies. Apart from the Futures Market, the SPOT market is the platform where most of the electricity volume is traded, what makes it the most significant and influencing the operation of the entire power system. The day-ahead market allows the suppliers and recipients to trade in the short term the excess/deficiency of energy which has not been included in future contracts. Knowing the future electricity prices allows to optimize the position of a given entity on the market, resulting in maximization of profits or minimization of losses. Inaccurate prediction may have a consequence in the necessity of additional purchases/sales on the balancing market, in which usually the prices are less profitable both from the suppliers/consumers/traders point of view.

As it was demonstrated in the section 3.2, the uncertainty of wind energy production plan is connected with considerable financial losses, because the privileged position of RES in the energy market has been abandoned.

The improvement of the SPOT market prices forecasts is of a high interest to the market players. Therefore, it has been decided to perform SPOT market prices forecasts by means of commonly used time series models, which have been studied in publications mentioned in section 2.1. Secondly, these models will be extended by addition of wind generation forecast as an external input in order to note whether this extension can bring and advantage in the obtained results. The simulation will be performed for both countries, what will answer the question whether the usability of predicted wind power is of the same importance in both Polish and Portuguese systems.

4.1. Persistence Model

Persistence models, also named as the naive models are characterized by their simplicity. Generally, the forecasted value of model takes the value of the last observation.

$$y_{t+1} = y_t \tag{16}$$

According to Sobczyk (2008), the naïve models can be used in short-term forecasting, when the constant systematic term can be distinguished and the random deviations are not substantial. The persistence model may be the best solution, when the time series describing the observed phenomenon is missing or is very short. Although it may seem to be fallible, the naïve models often perform comparably in reference to much more elaborate models [41].

4.2. Auto Regressive Moving Average Model (ARMA)

Box an Jenkins (1970) introduced a step-by-step methodology for modelling and estimation of time series by use of the autoregressive (AR) and Moving Average (MA) models. The commonness of this method for prediction purposes is supported by wide application in economy processes modelling, since many of them depend their state in the past. The current explained value is expressed by a combination of finite number of its past values, or/and interchangeably, their random disturbances [24].

The AR and AM methods, as well as their junction (ARMA) are basing on the autocorrelation within the observed time series, which is going to be explained in the further part of this section.

The autoregressive part of order *p* AR(p) is described by the equation 17 [42]:

$$y_t = \delta + \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \cdots + \emptyset_p y_{t-p} + \varepsilon_t \tag{17}$$

Where $\emptyset_p$ are the model parameters, δ is a constant term and ε_t is the random variable of mean value equal to 0, and constant variance σ^2(see Equation 3). The *p* value represents the number of past values of *y* taken into consideration in the model. Successively, the MA(q) model of order *q* is given by the formulae [42]:

$$y_t = \delta + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \tag{18}$$

where θ_q - parameters of *q*-order polynomial, with ε_{t-q} random error terms as explanatory variables. In order to improve the fitting of the model to the realization of explained variable, the combination of the aforementioned models (equations 17 and 18) is implemented and represented by the mixed model ARMA(*p,q*) [42]:

$$\begin{gathered} y_t = \delta + \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \cdots + \emptyset_p y_{t-p} \\ + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} \end{gathered} \tag{19}$$

The ARMA(*p,q)* model uses the p past values of the observed quantity and q values of random error terms of the time series *y*. Successful adoption of the ARMA model entails the necessity of fulfilling the particular requirements of the method, as well as following the 3-step procedure: model identification, estimation of the parameters and model verification. To properly identify the model, the examined time series has to meet particular requirements. First of all, it has to be stationary (there is no trend observable and the variance is constant). Once the stationarity is not confirmed, the time series has to be modified to achieve stationarity. Secondly, the parameters of the model have to be determined by mean of computational methods (e.g. maximum likelihood estimation or non-linear least-squares estimation). Further, the estimated parameters have to be verified statistically for proving their significance in the model. If the constructed model fulfils all the verification criteria, it can be applied for forecasting. Contrarily, if the evaluation of the model suggests that some of the parameters are not statistically significant or the model residuals manifest autocorrelation, the identification step has to be repeated and followed again until the model evaluation brings admissible results. The Box-Jenkins methodology is and iterative process, which graphically has been represented in the Figure 9 [42].

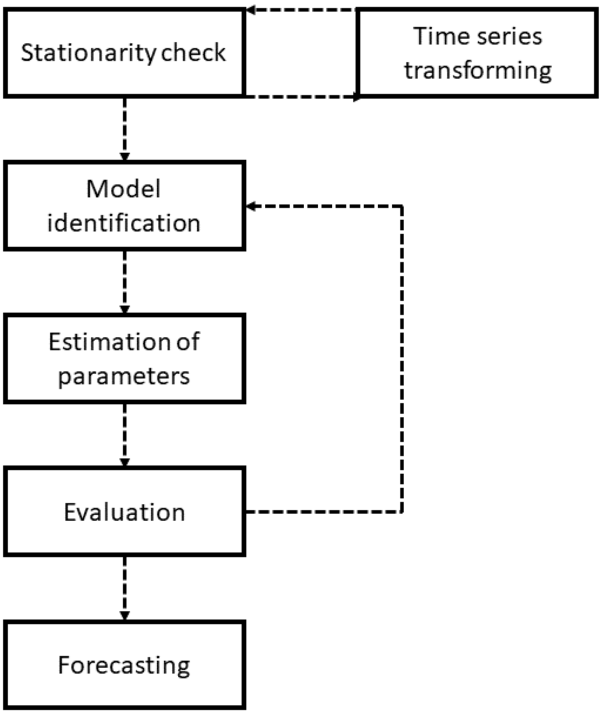


Figure 9 Box-Jenkins methodology flowchart

4.2.1. Verification of stationarity

As a fundamental feature of the times series subjected to ARMA models is its stationarity. The time series is considered as stationary if [42]:

a) The expected value (arithmetic mean) is constant along the entire observation
b) The value of the variance doesn't change along the entire studied time series

In some cases, the decision whether the observation is stationary or not can be made by analysing its plot visually. Nevertheless, sometimes this qualification has to be supported by other actions, for example, by analysing the autocorrelation function (ACF) of the series, which is going to be discussed in detail further in this section. Although, the proper interpretation of the autocorrelation function may require an experience of the model constructor [41].

Stationary time series should resemble the so-called "white noise" series of random errors ε_t, which perfectly fits to the ARMA model, because for all *t* [42]:

$$Ex(\varepsilon_t) = 0$$

$$Var(\varepsilon_t) = \sigma^2$$

If the process reveals lack of stationarity, it has to be modified. As one of the simplest methods for making the process stationary is differencing, which creates a new time series made of differences between two subsequent observations in the series [41]:

$$\Delta y_t = y_t - y_{t-1} \tag{20}$$

In execution of the first stage of Box-Jenkins methodology, the visual analysis of the autocorrelation function constitutes an initial step. By viewing the ACF function one may determine whether the process is stationary and non-deterministic (e.g. does not show periodicity). The ACF function can be understood as the series corresponding to correlation coefficients between the explained variable and its *k*-steps delayed realizations [27]:

$$r_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n}(y_t - \bar{y})^2} \tag{21}$$

Proper interpretation of the ACF functions helps to identify the most appropriate model structure. In the literature, numerous leads can be found for adequate concluding in analysis of the ACF [27]

Table 5 Leads for interpreting the course of ACF function [27]:

Course of the ACF function	**Interpretation**
Decreasing exponentially	AR part of the model is significant
Decreasing to zero with sinusoidal pattern	AR part of the model is significant
Several peaks observed, after which the sudden drop occurs	MA part of the model is significant
Initially constant values, then decaying to zero	Both MA and AR parts of the model are significant
Increments observed periodically	There exists seasonality. To use ARMA model, modify the time series to obtain white noise series
ACF values oscillate around zero	The series is random
The function does not decrease	Series is not stationary

It has been decided that the exemplary process of ARMA(p,q) model estimation will be performed in this subsection for better understanding of the Box-Jenkins methodology. The data used for estimation was the hourly time series of SPOT market prices from Jan – November of the year 2016, both for the Polish and Portuguese case.

The MatLab internal function `autocorr(ObservedSeries,numLags)` returns the autocorrelation function plot for the variable *y* and predefined number of lags to analyse (`numLags`). In the literature, this plot often is named correlogram. The number of lags to analyse has been arbitrarily assumed to be 100. The plots of the ACF functions has been shown in the Figure 10:

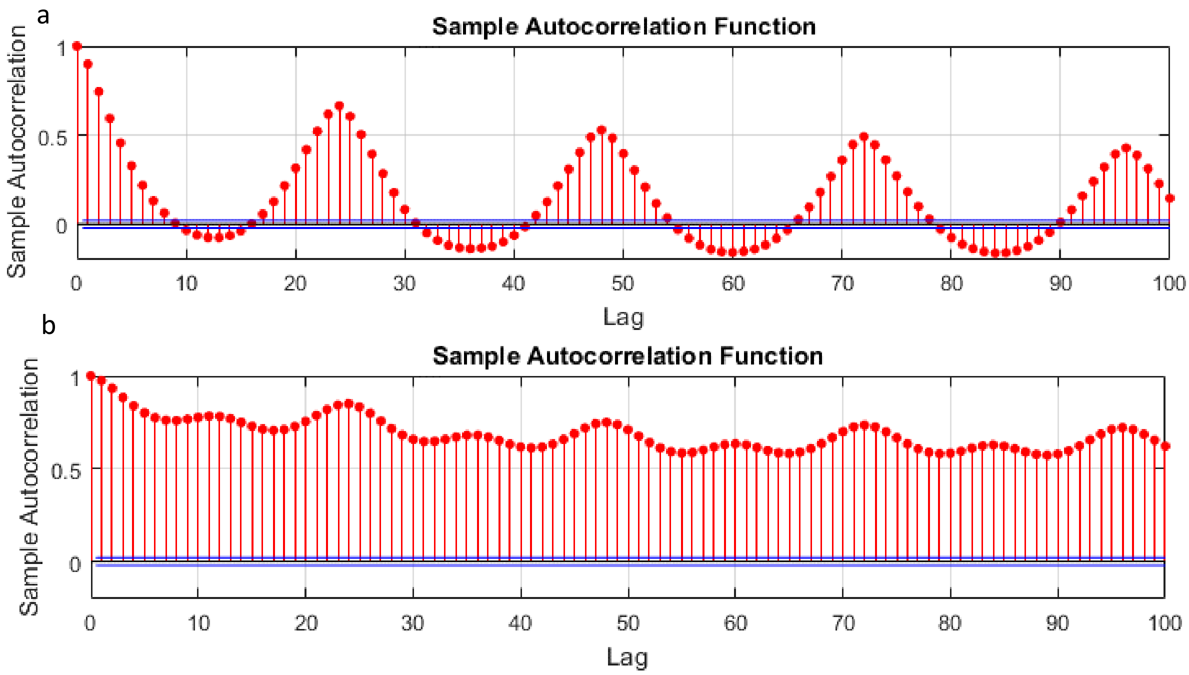


Figure 10 ACF function plots for electricity SPOT prices in Portugal (a) and Poland (b)

The blue lines on the chart represent the confidence intervals. The course of the plotted ACFs of the SPOT prices in both countries shows very slow decay, what, according to information in Table 4, suggests that the process is not stationary. Additionally, the considerable periodicity has been observed, being correspondent to the lag *k* equal to 24 hours.

Following the Box-Jenkins algorithm (see Figure 9), detection of non-stationarity requires the modification of the time series. In order to do so, the analysed time series has been subjected to differencing, called also integration (Bielińska, 2007). Although, to remove the periodicity of the series, the SPOT prices series have been differenced by time-lag of 24 hours:

$$\Delta y_t = y_t - y_{t-24} \tag{22}$$

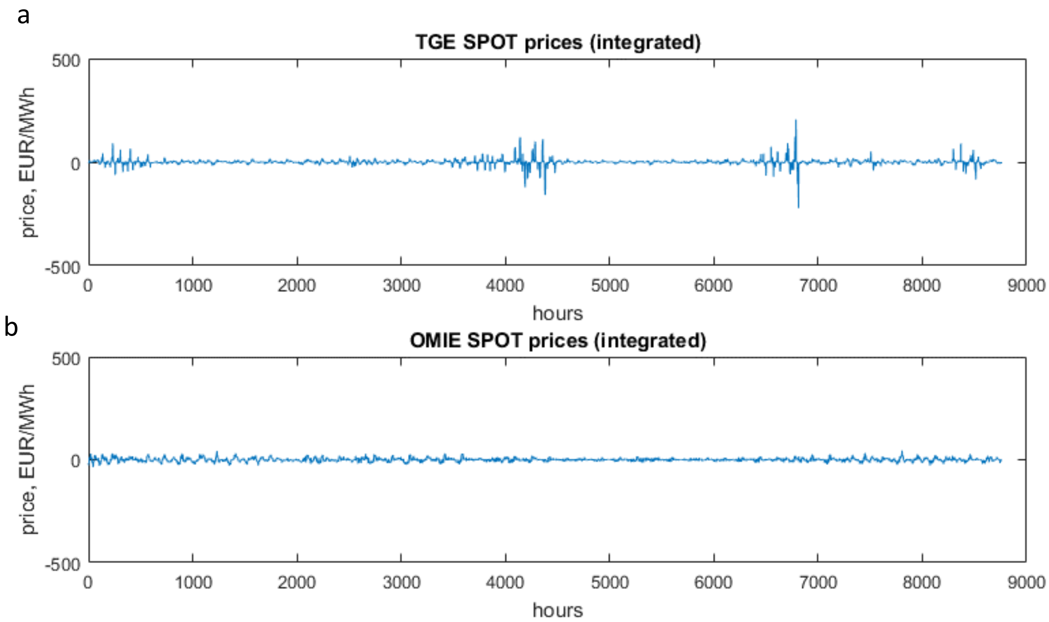


Figure 11 Time series plots of integrated electricity SPOT prices in Portugal (b) and Poland (a)

In the Figure 11 the integrated time series of the SPOT market prices have been shown for both countries. Simply speaking, from this point, the analysed time series consists of the results of subtraction of the SPOT price at time (t-24) from the price at time t. Analysing the Figure 11, the oscillation over 0 value can be observed, with no trend at the same time and with relatively constant volatility of the time series (white noise properties), which, however, is more noticeable in the case of TGE case. In the Figure 12 the ACF function plot have been presented for the integrated time series of SPOT market prices.

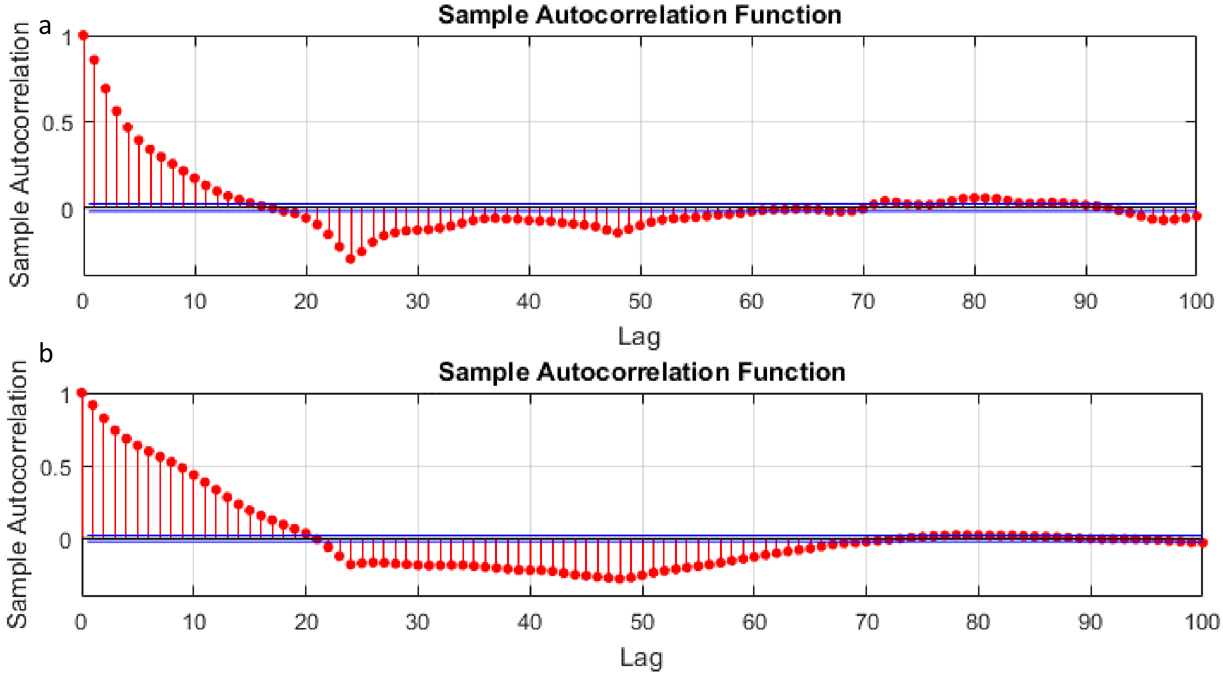


Figure 12 ACF function plots for electricity SPOT prices in Portugal (a) and Poland (b)

By analysing the Figure 12 it can be observed that considerable autocorrelation occurs for lags up to 3 in case a (Portugal) and up to 5 in case b (Poland), reaching markedly the values above 0.6 . The courses of the ACF functions for integrated processes in both cases reveal diminishing pattern. Summing up the outcomes of the performed integration, the modified time series can be considered as stationary, what allows to follow the next steps in the Box-Jenkins methodology.

4.2.2. ARMA(p,q) polynomial orders.

Besides the verification of stationarity of the series, the identification of the model requires also determining the orders p and q or AR and MA polynomials, respectively. One of the approaches to do this is to analyse the ACF plots together with PACF (Partial AutoCorrelation Function) and detect the number of lags which indicate the p and q values. The difference between the ACF and PACF comes from elimination of the influence of intermediary observations (from *k*=1 to *k*-1) [42].

$$r_{kk} = \begin{cases} r_1 & if\ k = 1 \\ \dfrac{r_k - \sum_{j=1}^{k-1} r_{k-1,j} \cdot r_{k-j}}{1 - \sum_{j=1}^{k-1} r_{k-1,j} \cdot r_k} & if\ k = 2,3,\ldots \end{cases} \tag{23}$$

Analysis of the PACF function may help only in determining the AR(p) model order (Bielińska 2007). The Matlab software includes also the tool for plotting PACF function `parcorr(y,numLags)`. The plots of ACF function for integrated SPOT prices time series have been already shown in the Figure 12, while the PACF function courses for these series has been pictured in the Figure 13.

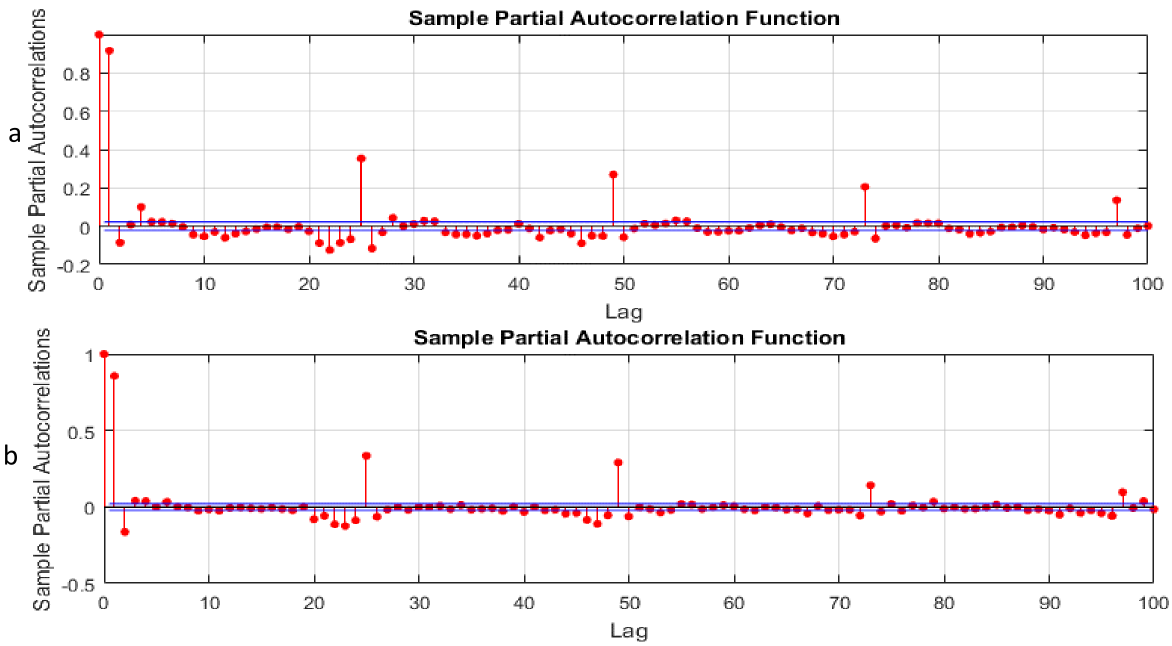


Figure 13 PACF function plots for integrated series of electricity SPOT prices in Portugal (a) and Poland (b)

When analysing the PACF plots, one may observe that in both cases, the only significantly outstanding lag is 1, optionally 2. Although there can be observed some partial autocorrelation for the lags 24 in both cases, their inclusion in the model would require inclusion of all the intermediate lags, which will have no valuable information to the model. Unfortunately, the Matlab AR function term does not allow to choose one particular lag (e.g. k=24) without considering remaining 23 in the polynomial model.

In the literature emphasis is put on the difficulty of proper interpreting the ACF and PACF functions when not having extensive experience. For this reason, the ultimate orders of the ARMA model are going to be found by means of iterative methods, due to the accessibility of computational software. The ranges of polynomial orders will constitute a domain for finding the ultimate, final polynomial orders in the next section. For every combination of the p and q values, the model parameters are going to be estimated by maximizing the likelihood function L (see section 3.4.3). The best model is characterized by the highest L function value for corresponding combination of p and q. Although, it may occur that the best fit is obtained for very high-order polynomials, what is burdened by considerable computing time. Tough, the Akaike Information Criterion (AIC) has been introduced, which indicates the most accurate model with implementing the penalty for complexity (number of coefficients to estimate) [42]:

$$AIC = -2\log(L) + 2k \tag{24}$$

where L stands for Likelihood function value and k for total number of coefficients to estimate. Alternatively, one may use the Bayesian Information Criterion (BIC), which is penalize the complexity to a greater extent:

$$BIC = -2\log(L) + klog(N) \tag{25}$$

where N represents the sample size. Contrarily to the selection of the model by finding the highest value of L, the chosen model should have possibly the lowest value AIC or BIC.

4.2.3. Estimation of ARMA(p,q) parameters

Once the structure of the model is determined, the next step requires fitting the coefficients of polynomials shown in Formulae 21. As one of the most common methods for their estimation is the maximization of likelihood (Box and Jenkins, 1994). For the whole set of observation, the likelihood function L is defined, which reflects the probability of obtaining the model outputs exactly equal to the actual observations. Maximization of the likelihood function brings the values of maximum likelihood estimators, which implemented in the model give the highest probability of obtaining perfect fit of the model [42]. Finding the optimal values of parameters is most frequently conducted via iterative way, by means of computational techniques, which are going to be applied in this study – namely, internal Matlab functions. The essentials of the Matlab code generating a model structure, estimating polynomial parameters and calculating the AIC information has been shown in the Appendix C, together with explanatory comments.

Reassuming, for each pair of p and q (12 combinations available), the model is created, the parameters are estimated and AIC function calculated, which is prior criterion in selecting the p and q values. The Table 5 shows estimation results with highlight on the one with best performance of aicbic Matlab function. The same iterative process has been made for Polish and Portuguese SPOT prices integrated time series, respectively. The noticeable by the eye differences between the shape and course of analysed time series (see Figure 11) suggest to expect different outputs of the model identification process. The table on the next page shows the results of iterative process of model identification:

Table 6 Extracted results of iterative search for the most information-contained ARMA (p,q) model structure

TGE price SPOT time series					OMIE price SPOT time series				
p order	**q order**	**log(L)**	**AIC**	**BIC**	**p order**	**q order**	**log(L)**	**AIC**	**BIC**
0	0	-31550.2	63104.42	63118.40	0	0	-28183.5	56370.91	56384.89
0	1	-28369.3	56744.53	56765.50	0	1	-24705.3	49416.60	49437.57
0	2	-26937.7	53883.38	53911.34	0	2	-22853.2	45714.40	45742.36
0	3	-26467.8	52945.62	52980.57	0	3	-22040.2	44090.39	44125.33
0	4	-26211.0	52433.99	52475.93	0	4	-21635.7	43283.31	43325.25
0	5	-26095.3	52204.68	52253.61	0	5	-21401.7	42817.39	42866.31
0	6	-26026.8	52069.55	52125.46	0	6	-21227.9	42471.85	42527.76
1	0	-26078.5	52163.05	52184.01	1	0	-20828.0	41662.03	41683.00
1	1	-25931.7	51871.36	51899.31	1	1	-20803.3	41614.67	41642.62
1	2	-25927.4	51864.74	51899.69	1	2	-20801.0	41611.91	41646.85
1	3	-25916.4	51844.89	51886.83	1	3	-20763.8	41539.59	41581.53
1	4	-25916.3	51846.63	51895.55	1	4	-20760.4	41534.83	41583.75
1	5	-25913.6	51843.15	51899.06	1	5	-20760.3	41536.52	41592.43
1	6	-25913.5	51844.99	51907.89	1	6	-20759.5	41536.98	41599.88
2	0	-25932.1	51872.25	51900.21	2	0	-20802.4	41612.77	41640.73
2	1	-25929.1	51868.13	51903.08	2	1	-20802.4	41614.72	41649.66
2	2	-25919.0	51850.06	51891.99	2	2	-20792.2	41596.30	41638.24
2	3	-25915.8	51845.62	51894.55	2	3	-20760.1	41534.26	41583.19
2	4	-25915.6	51847.24	51903.15	2	4	-20760.0	41536.10	41592.01
2	5	-25913.5	51845.02	51907.93	2	5	-20693.8	41405.57	41468.47
2	6	-25913.2	51846.31	51916.20	2	6	-20691.8	41403.54	41473.43

In the above Table, the rows highlighted by blue colour correspond to the best performance model with regard to the AIC criterion. On the other hand, the yellow rows contain the lowest values of BIC criterion. Generally, both selection approaches point in the similar model structures. The final selection has been made basing on the BIC criterion, which puts more focus on the complexity (number of unknown parameters). Summarizing, the joined procedures of model identification and estimation of its parameters gave most information-contained structures: ARMA(1,3) for TGE and ARMA(2,5) for OMIE SPOT prices series, which would be subjected to evaluation procedure in the next subsection.

4.2.4. ARMA(p,q) model evaluation

In the literature, numerous methods for evaluation of the models can be found. One of the most commonly used is the t-test, which allow to verify the statistical significance of each of the estimated parameters in the model. This verification is based on testing the null hypothesis, equal to 0, stating that the considered term (here polynomial coefficient) is insignificant versus the alternative one (term is different from 0) [43].

$$H_0 : \beta_i = 0$$

$$H_1 : \beta_i \neq 0$$

The acceptance/rejection of the null hypothesis can be made by comparing the t-test value with the critical value from the standardized t-test table. The considered term is statistically significant, if its t statistic value exceeds the critical value of t statistic for particular number of degrees of freedom (here: N-2) and confidence level, most frequently set to 95%. The results of the t-test have been shown for the models with the best BIC performance (Table 7).

Table 7 t-test results for the ARMA model parameters

TGE SPOT price time series				**OMIE SPOT price time series**			
Parameter	Value	Standard error	t-Statistic	Parameter	Value	Standard error	t-Statistic
AR{1}	0.830	0.004	194.078	AR{1}	1.941	0,004	521.908
MA{1}	0.202	0.005	38.652	AR{2}	-0.944	0,004	-253.282
MA{2}	-0.008	0.005	-1.983	MA{1}	-0.972	0.010	-102.178
MA{3}	-0.068	0.005	-12.406	MA{2}	-0.071	0,014	-5.184
				MA{3}	-0.094	0.012	-7.637
				MA{4}	0.095	0.013	7.367
				MA{5}	0.053	0.011	4.756

For the number of degrees of freedom equal to 8014 (8016 observations in the learning dataset) and confidence level of 95%, the critical value of t-student statistic is equal to 1.96 [24]. Thus, the t-statistic values of estimated model's coefficients should take the value higher than 1.96. Comparing the values of t-statistic with the critical one, the conclusion is that all the estimated coefficients for both TGE and OMIE cases are statistically significant to the constructed model.

4.3. Auto Regressive Moving Average model with External Input (ARMAX)

The idea which stands behind the ARMAX model resembles the model construction process described in subsection 3.4, but with inclusion of the additional variable in the model. Hitherto, the ARMA model used only the lagged observations of explained variable. In the ARMAX approach, the additional information as the external variable can be provided to the model. In this case, the ARMA model expression described in Formulae 21 is extended in the following way [44]:

$$\begin{aligned} y(t) + a_1 y(t-1) + \cdots + a_{na}(t-n_a) \\ = b_1 u(t-1) + \cdots + b_{nb}(t-n_b) + \varepsilon(t) \end{aligned} \tag{26}$$

$$+c_1\varepsilon(t-1)+\cdots+c_{nc}\varepsilon(t-n_c)$$

where: a_{na}- coefficients of the AR part of the model ($\emptyset$ in ARMA model)
c_{nc}- coefficients of the MA part of the model (θ in ARMA model)
b_{nb}- coefficients of the X part of the model
n_a, n_b, n_c – polynomial orders

Formulae 28 can be also expressed in more compact, matrix form [44] :

$$A(z)y(t)=B(z)u(t)+C(z)e(t) \tag{27}$$

where the capital letters represent the matrices of coefficients of polynomials of each integral part of the model [44]:

$$\begin{aligned} A(q) &= 1+a_1z^{-1}+\cdots+a_{na}z^{-na} \\ B(q) &= b_1z^{-1}+\cdots+b_{nb}z^{-nb} \\ C(q) &= c_1z^{-1}+\cdots+c_{nc}z^{-nc} \end{aligned} \tag{28}$$

where the *z* – lag operator is introduced, such that [44]:

$$y_tz^{-k}=y_{t-k} \tag{29}$$

Then, the set of coefficients to determine is:

$$[a_1\ a_2\ \cdots\ a_{na}\ b_1\ b_2\ \cdots\ b_{nb}\ \ c_1\ c_2\ \cdots\ c_{nc}\ ]$$

Despite the fact that the general principle of the ARMA and ARMAX models is similar, the algorithms used for their estimation differ markedly when using Matlab as an estimation tool. In creating the ARMA model, the series of subsequent steps (and corresponding functions) had to be executed in order to obtain the estimates of polynomial coefficients. In the case of ARMAX model preparation, the function `armax(data,[na nb nc nk])` creates the model, as well as iteratively estimates the most robust and credible coefficients by means of "*robustified quadratic prediction error criterion*" (Mathworks, 2017).

What also should be noted is the data input character – in the ARMA model case, all the operations have been made on double type vectors while the usage of ARMAX method in Matlab required converting the data into `iddata`, time-domain variable type [44].

The evaluation of the ARMAX model can be carried out in the same way as in the case of ARMA model, what has been exhaustively described in the section 4.2.

4.4. Nonlinear Auto Regressive ANN Model

The MatLab includes a wide range functions dealing with construction, training and simulation of the ANNs. For the purposes of this study, the specific case of time series ANN models are going to be examined as an alternative for the ARMA and ARMAX methods described in the former in previous subsection. The general idea of the NAR model resembles the concept of the ARMA model (Figure 14).

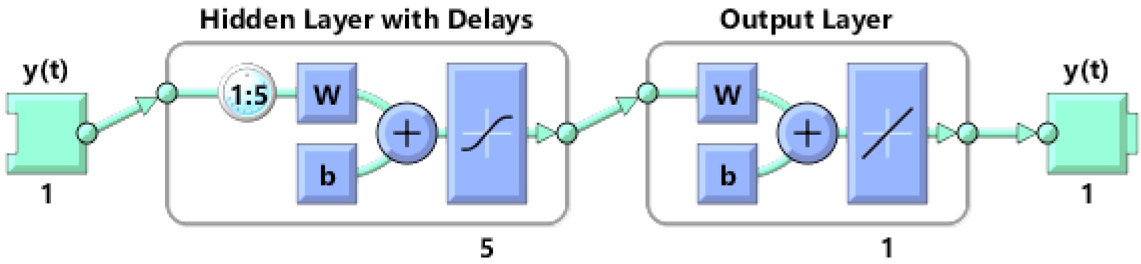


Figure 14 Graphical representation of NAR model – 5 input variables and 5 hidden layers

The model output is determined basing on the previous values of the observed quantity. However, the modelling process id of a diametrically different character.

$$y(t) = f(y(t-1), y(t-2) \ldots y(t-5))$$

In the above figure, a representative example of the NAR model has been shown for the ANN with 5 input variables (here: 5 past values of the observation *y*). **b** letter stands for bias, while **w** represents the weights. A brief explanation of the key ANN issues has been explained in the succeeding subsection.

4.4.1. Basic concepts

Artificial Neural Networks (ANN) in principle of operation resemble human brain. The origins of the ANN took place in 1943, when the first Artificial Neuron model has been introduced by McCulloch and Pitts. Contemporarily, ANN are commonly used in statistics and in signal processing. Despite the advanced development of this branch of science, the range of ANNs application is continuously expanding [47].

The Neural Network consists of neurons, which are simple processors arranged in layers. The signals are processed and sent further to the succeeding layers, in which the task to solve is gradually simplified. Thanks to the parallel data processing within the layers, the set of primitive neurons constitute the learning-capable structure. Often, in the ANN nomenclature the word "neuron" is replaced by "unit". The units are interconnected by links, which are attributed by particular numeric weight. The standard structure of the ANN consists of input units (vector of numbers provided by the user), hidden units (representing the intermediate calculations) and output units (vector of the model results) [47].

In the literature, two main ANN structures can be pointed out: recurrent and feed-forward nets [47]. The former one assumes the versatility of the connection arrangements (the topology is arbitrary), while the latter structure is restricted by only one direction of the links. The special case of the feed-forward ANN with no hidden layers is named perceptron. The graphical representation of a basic ANN unit and feed-forward network structure have been shown in the Figure 15.

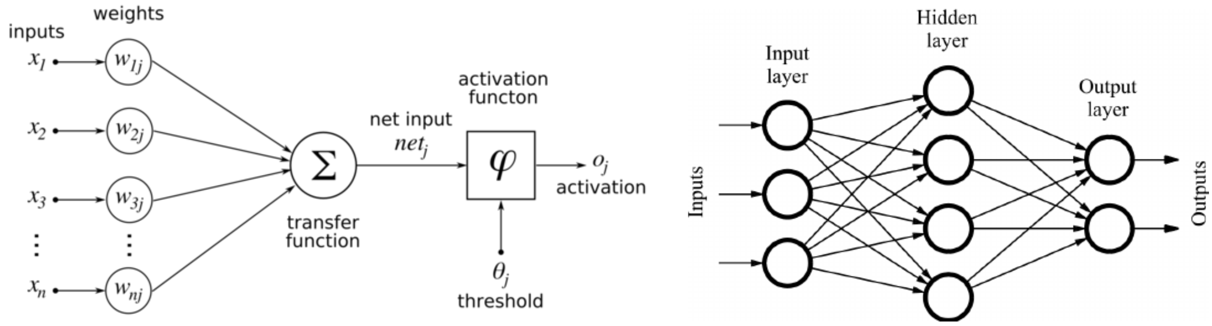


Figure 15 Graphical representation of the ANN unit and feed-forward ANN [47]

Where the *x* corresponds to the input signals to the network, while *w* is a weight assigned to particular input signal. Transfer function φ sums up the weighted signals (generates net input). The weights reflect the importance of a given signal in the model – the higher is the weight value, the more significant is the input signal in the process of output determination. Therefore, the weights can take both negative or positive values. The weights adjustment constitutes the "learning" process of the network. In the process of the network building, the following steps have to be made: (i) setting the number of units contributing in the network, (ii) determining the type of units (iii) defining the connection type between the units [47].

There are several approaches to the input signals processing by the activation function, which can be divided in two main forms: linear and non-linear. In the linear method, the weighted sum of the input signals is added to a so-called bias (threshold), which is a number with and individual weight as well, what gives the output signal to the succeeding layers. On the other hand, among the non-linear activation functions the most commonly used are: step function or sigmoid function [47].

The step function returns a certain value, if the input weighted sum exceeds the threshold value. Contrarily, if the summarised input doesn't take the value above the threshold one, the step function returns different value. In the basic perceptron, the output values are 1 and 0, for the case of threshold exceeded and not, respectively. The sigmoid function (called also logistic function) can be expressed by the following equation:

$$\sigma(t) = \frac{1}{1 + e^{-\beta t}} \tag{30}$$

Where t represents the input value and β is a certain parameter influencing the shape of the function. The graphical representations of the step and sigmoid functions have been shown in the figure below.

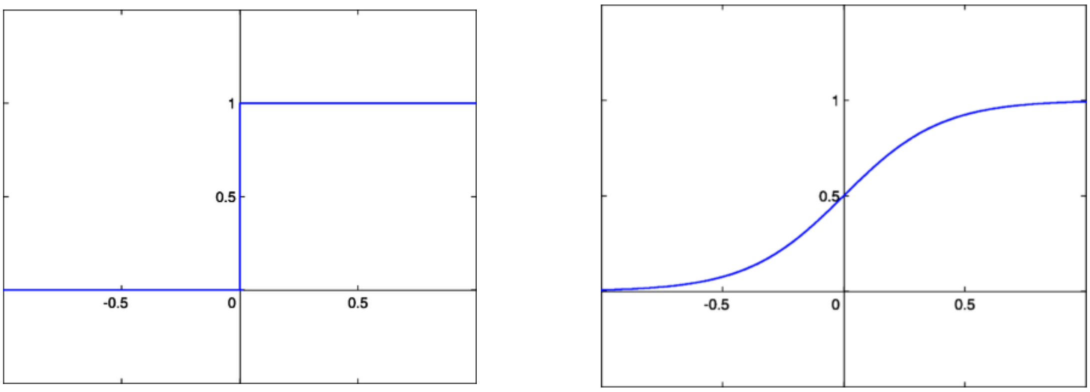


Figure 16 Step function and sigmoid function graphical examples [47]

Besides the definition of the network architecture and selection of the activation function, the choice of network training algorithm has to be made as well in order to make the model complete. In general, the main goal of training algorithms is to set the values in the weights vector in the way allowing to obtain a minimum of a loss function, which reflects the quality of the model-to-reality fit. Since in most of the cases, the ANN models regard multiple variables, the training algorithm becomes a task of multidimensional optimization. The ANN learning algorithms are based on the iterative processes, where the inaccuracy of the model is mitigated gradually after each sequential step. Most often, the algorithm is stopped after satisfying so-called *stopping criterion*, which for instance may be a satisfactory value of the model error. Among many training algorithms available to apply in ANN, below, the most recognizable ones have been listed with the most important characteristics [49]:

- Gradient descent – is featured by high simplicity, bases on the detection of the steepest decrease of the loss function by means of gradient. The size of the step made by the algorithm is training rate, which can be a fixed number or be internally optimized on the way of single iteration. As the main disadvantage of this method, the convergence-time is highlighted, especially for the loss functions with relatively slight fall. On the other hand, it is recommended for the nets dealing with numerous variables, because it does not require a lot of computing memory.
- Newton's method – in contrary to Gradient descent (first order - gradient method), the Newton's method bases on the second order derivatives (Hessian matrix, explained further). The application of Hessian matrix allows to determine the searching directions more

accurately. This method requires considerably less steps to find a minimum in comparison with gradient method, however, requires much more computational effort, what is influenced by matrix operations.

- Conjugate Gradient – Can be considered as the improved method of the gradient descent method, because the achievement of optimum is made in a faster way, keeping the simplicity of calculations on the unchanged level.
- Levenberg-Marqardt – in principle, it works similarly as the Newton's method, although, the Hessian (second order derivatives) matrix is approximated by Jacobian matrix. The Jacobian is a matrix composed of first-order derivatives of the loss function with respect to the parameters of the network. The training steps are influenced by damping factor. The limitation of this method is that it cannot be applied to any kind of loss function. Despite the approximation of the Hessian matrix by the Jacobian one, it still requires significant computational capabilities. For the purposes of the thesis, the LM algorithm will be used in modelling the SPOT prices – therefore, it has been explained more explicitly in the subsection below.

4.4.2. Levenberg-Marqardt training algorithm [45] [47].

The Levenberg-Marquardt (LM) algorithm is one of the most commonly used NN learning techniques. The weights modification by means of this method is made in a group manner, it is after providing all the learning vectors. It is characterised by high effectiveness in feedforward networks training, and combines the convergence of the Gauss-Newton algorithm with the fastest decline methods.

In comparison with the conventional BP method, the LM algorithm features fast operation with simultaneous increased memory demand [45].

The LM training algorithm is based on the method for finding the roots of the Newton function. For the loss function *f(x),* the derivative in point x_0 is defined, presented graphically in the Figure 17.

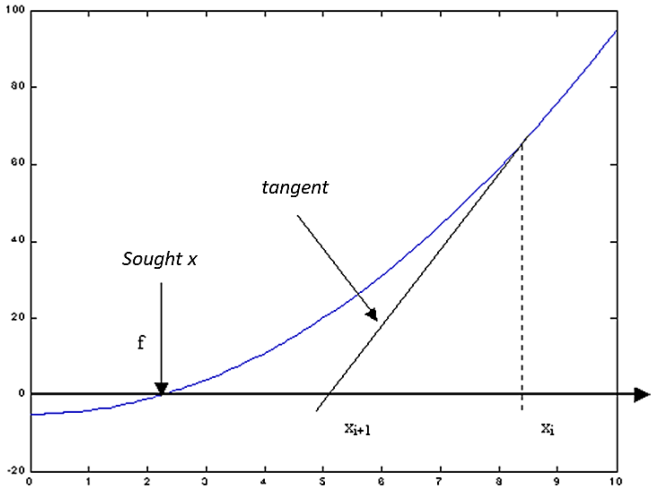


Figure 17 Graphical representation of the Newton function and its derivative at x_0 [47]

The derivative of the function *f* at x_0 presented in the above figure can be expressed as the tangent of the created rectangular triangle:

$$tg(\alpha) = \frac{f(x_i) - 0}{x_i - x_{i+1}} = f'(x_i) \tag{31}$$

Thus

$$\frac{f(x_i)}{x_i - x_{i+1}} = f'(x_i) \tag{32}$$

By modification of the equation 32, the iterative step of the Gauss-Newton method for finding the roots can be expressed as follows:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{33}$$

In this case, finding minimum is related with finding the root of a derivative. If denoting *f* by *g'*, the Formulae 33 takes the following form:

$$x_{i+1} = x_i - \frac{g'(x_i)}{g''(x_i)} \tag{34}$$

When considering the Newton's minimisation for the *n*-dimensions, the equation 34 is globally represented by the formulae 35:

$$W_{i+1} = W_i - H^{-1}(W_i)\nabla g(W_i) \tag{35}$$

where: $H^{-1}(W_i)$ - Hessian matrix, which corresponds to the *g''* derivative from the Formulae 34,

$\nabla g(W_i)$ – gradient, which corresponds to the g' derivative from the Formulae 34.

$$H_{ij} = \frac{\partial}{\partial w_i}\frac{\partial}{\partial w_j} g \tag{36}$$

The LM method is based on finding the direction of a vector p:

$$p = -(J^T J + \mu I)^{-1} g \tag{37}$$

Were J – Jacobi matrix, μ – regulation parameter. The Jacobi matrix and *g* vector are dependent on the network error *E(w)*:

$$e(W) = \begin{bmatrix} e_1(W) \\ e_2(W) \\ \vdots \\ e_M(W) \end{bmatrix} \qquad e_i = [y_i(W) - d_i)] \qquad E(W) = \frac{1}{2}\sum_{i=1}^{M}[e_i(W)]^2 \tag{38}$$

The Jacobi Matrix is then expressed as in Equation 39:

$$J(W) = \begin{bmatrix} \frac{\partial e_1}{\partial W_1} & \frac{\partial e_1}{\partial W_2} & \cdots & \frac{\partial e_1}{\partial W_n} \\ \frac{\partial e_2}{\partial W_1} & \frac{\partial e_2}{\partial W_2} & \cdots & \frac{\partial e_2}{\partial W_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial e_M}{\partial W_1} & \frac{\partial e_M}{\partial W_2} & \cdots & \frac{\partial e_M}{\partial W_n} \end{bmatrix} \tag{39}$$

The *g* function can be defined in short form:

$$g = J^T e \tag{40}$$

The Hessian matrix is approximated according to the Newton's method:

$$H \approx J^T J \tag{41}$$

In the next step, the μI matrix is added in order to obtain a positively-defined matrix. The LM algorithm begins with setting relatively high values of the damping coefficient μ, which is decreased every time when the obtained results indicated improvement (Newton method), or increased in the case of obtaining higher error values (gradient descent method). In simplified form, the LM algorithm is a set of the following actions [45]:

1. Set the initial values of the μ parameter
2. Provide the net with the vectors of learning data, calculate the net outputs, calculate the net error (38)
3. Calculate the Jacobi matrix (39)
4. Calculate new weight by means of the equation (37)
5. Calculate the net output by means of the learning dataset and newly defined weights
6. If the algorithm brought smaller error, the weights values should be kept and used again in the point 2 with decreased μ damping coefficient. If the results have worsened, the μ damping coefficient should be increased and the calculation should be repeated from the point 4.

7. The LM algorithm finds the solution, if the obtained error decreased, compared to previously determined value.

The inconvenience of the LM method is that it works in a local manner, what means that there is no warranty of finding the global minimum of the goal function. As an additional disadvantage one may mention the computing memory demands – the LM method requires calculation and inversion of the Jacobi matrix of the error function. The dimensions of the Jacobi matrix are determined by the number of weights in the entire net. Although, this requirement is compensated by fast convergence rate [45].

For the purpose of this study, the `trainlm` Matlab function will be used for training the ANN time series model for electricity SPOT prices forecasting.

4.5. Nonlinear Autoregressive ANN Model with External Input (NARX)

The MatLab's Naural Nets Library allows to choose the specific model type from many available. The NARX model differs from the NAR described in the section 3.6 by the availability of adding the external, independent input variable time series to the model. This solution could be interpreted as the non-linear correspondent of the ARMAX model discussed in section 3.5. The principles , requirements and mechanisms of the ANNs described in the section devoted to NAR model apply in this case as well, with one distinction, which is the additional external input, what has been illustrated in the Figure 18.

$$y(t) = f(y(t-1), y(t-2) \dots y(t-5), x(t-1), x(t-2) \dots x(t-5)$$

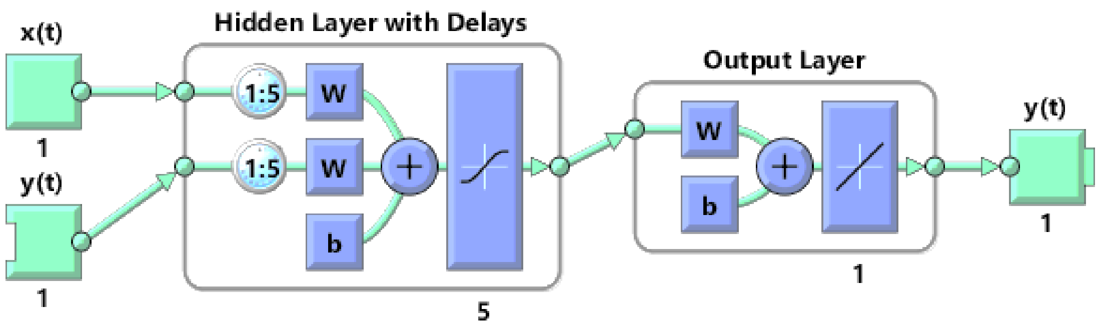


Figure 18 Graphical representation of NARX model – 10 input variables and 5 hidden layers

Summarizing, the certain number past values of the **x(t)** and **y(t)** constitute the input to the ANN model, which further are weighted and processed in the hidden layers of the net, undergoing the training algorithm (in the case of this study – LM algorithm).

5. Results and discussion

5.1. Available data

The source of the data for SPOT prices forecasting is the ENTSOE were the numerous data about energy systems and markets is continuously updated. As the mission, the ENTSOE aims "to develop with the energy transition and the successes of European market integration" [38]. As one of the implementations of the mentioned goals, the transparency of the integrated energy systems is maintained by publishing up-to date energy systems data, what this study made use of.

The data processed in this study constitute 4-time series: energy SPOT prices and day-ahead forecast of wind energy injected to the system– for both Portugal and Poland. The time range of the data is the entire calendar year 2016 with the fragmentation of 1 hour, what gives 8784 observations of each of the time series. To enable the direct comparison of the results, the TGE SPOT Polish market prices originally given in PLN currency were recalculated by the actual for a given hour EUR/PLN ratio given by the European Central Bank, what required the acquisition of the correspondent time series. In the Figure 19, the time series of wind generation forecasts have been shown for the calendar year 2016 in hourly fragmentation.

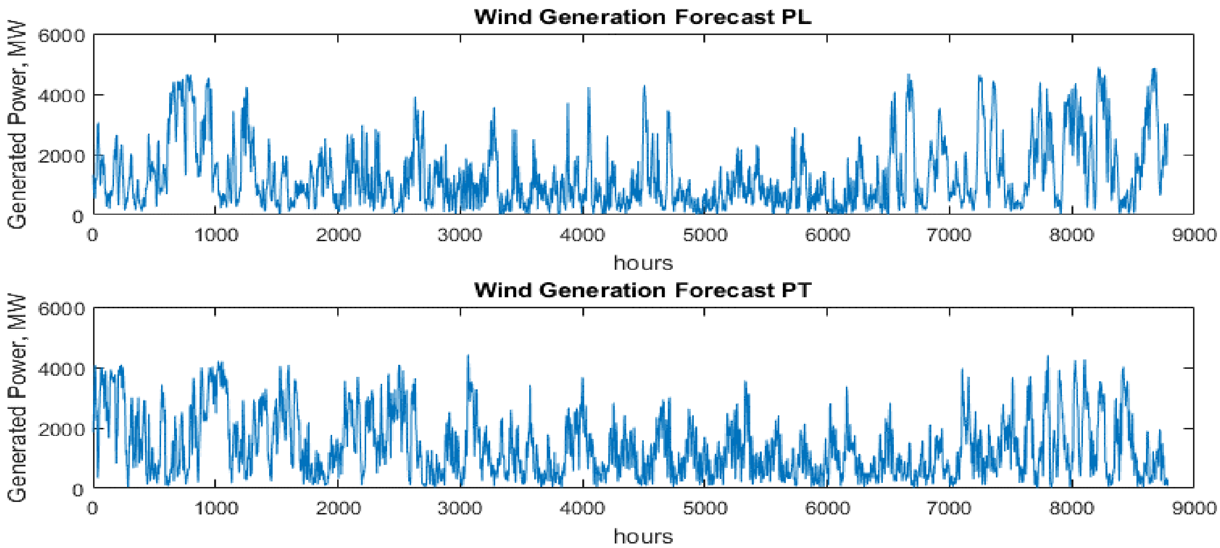


Figure 19 Wind generation time series in Poland and Portugal for the calendar year 2016

The graphical representation of the wind generation forecasts reveals the irregularity of the observed quantities. Determination of a seasonal or periodical pattern is impossible; the process is characterised by randomness. On the other hand, the visualization of the SPOT prices in the same time domain allows to observe a regular oscillations of the market price values along a time. Additionally, the appearance of incidental peaks in values has been detected (Figure 20).

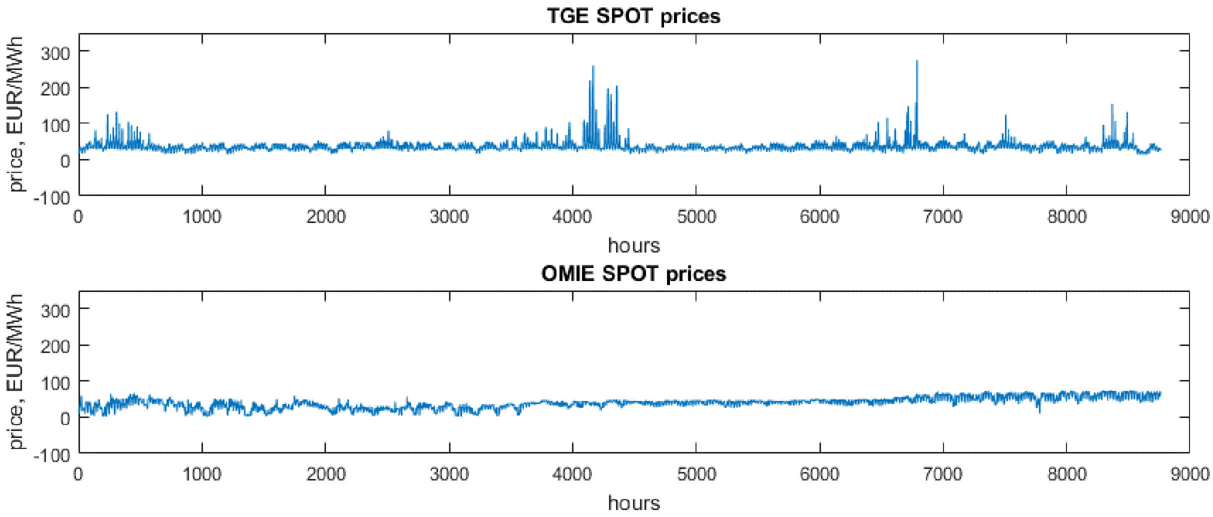


Figure 20 TGE and OMIE SPOT prices in Poland and Portugal for the calendar year 2016

Comparing visually both SPOT prices series, the Poland energy price denotes high volatility, while the electric energy prices in Portugal are more clustered around a constant value. Moreover, a slight increasing trend is noticeable in the OMIE energy prices, especially in second half of the analysed year.

Since the time series reveal periodicity, what eliminates the direct application of ARMA and ARMAX models, all the forecasts will be made basing on differenced(integrated) time series by 24 hours lag (see Figure 11, Equation 22).

To make all the models constructed in this study comparable, the division of the data has to be consistent. For this purpose, the time series has been decomposed into two sets: learning (from January until November) and validating (December). Following this approach, the forecasts are going to be estimated 1 step (1 hour) ahead for the total time-horizon of the entire December 2016 (744 hours)

5.2. Forecast evaluation criterion

As a measure of the forecasting quality, the Mean Absolute Percentage (MAPE) error is going to be used. The MAPE expresses the relative and absolute percentage deviations of the forecasted value from its realization, referred to the entire evaluated population [40]:

$$MAPE = 100\% \cdot \frac{1}{N}\sum_{t=1}^{N}\frac{|y_t - y_t^*|}{y_t} \tag{42}$$

where y_t, y_t^* represent the values of the actual and forecasted quantity, respectively.

For having a perspective on the financial burden correspondent to the inaccuracy of the performed forecasting models, the total uncertainty of sales of the wind energy in December 2016 (744 hours) has been calculated according to the equation 43, under assumption that the entire wind energy is traded on the SPOT market:

$$SU = MAPE \cdot \sum_{t=1}^{744} E_t^* \cdot M_t \tag{43}$$

Where SU – total sales uncertainty [EUR], E_t^*– traded wind energy volume in an hour t [MWh], M_t – SPOT market price in an hour t [EUR/MWH].

5.3. Forecasting approaches – case studies

In this part, the forecasting results of the models presented in section 3, designed by means of the MatLab software are going to be presented. On the way of study of the six selected models (section 3) it occurred, that in the case of ARMA and ARMAX models there is a methodology which allows to determine the best model structure, containing the most valuable and reliable information (e.g. by use of Akaike criterion, t-statistic). In the case of ANN models, there is no universal recipe for finding the best net architecture, resulting in most accurate results. Therefore, to make the prediction evaluation as uniform as possible, it was decided to find the best structures in the iterative way for all the six models used in this study, by means of the MAPE as evaluation criterion. This means that the most accurate model structures will be found by running the forecasts iteratively for:

a) varying *p* and *q* (ARMA) and varying n_a, n_b, n_c (ARMAX) polynomial orders
b) varying number of lags and hidden layers in the NAR and NARX models

The representative result of a particular model will be the one characterised by the lowest value of MAPE in the period of December 2016.

The external time series, used in ARMAX and NARX models contains the overall wind power generation prediction time series with the same time-fragmentation as the SPOT prices time series. In order to answer the main question of this thesis, which is the influence of wind generation forecast on the SPOT market prices, it will be verified, whether this information as input to the extended models can be valuable in terms of improvement of the time series SPOT prices forecasts, what would prove the relation between these two quantities.

Using the computational capabilities of the MatLab software and the possibility of looping of procedures executed in terms of particular projects, it was decided that the forecasting will be made by using all six models in the iterative manner, also with additional distinction on the training data set: static and dynamic. The static approach bases on a singular estimation of the model's parameters which are used in unchanged form for the entire prediction horizon (here - 1 month). On the other hand, in the dynamic approach, the model's parameters are updated every prediction step, basing on modified learning dataset, which is shifted also by one step towards, respectively.

For making the above clearer and more understandable, the Figure 21 shows the division of the forecasts, which will be carried out both for Poland and Portugal in breakdown into particular study cases.

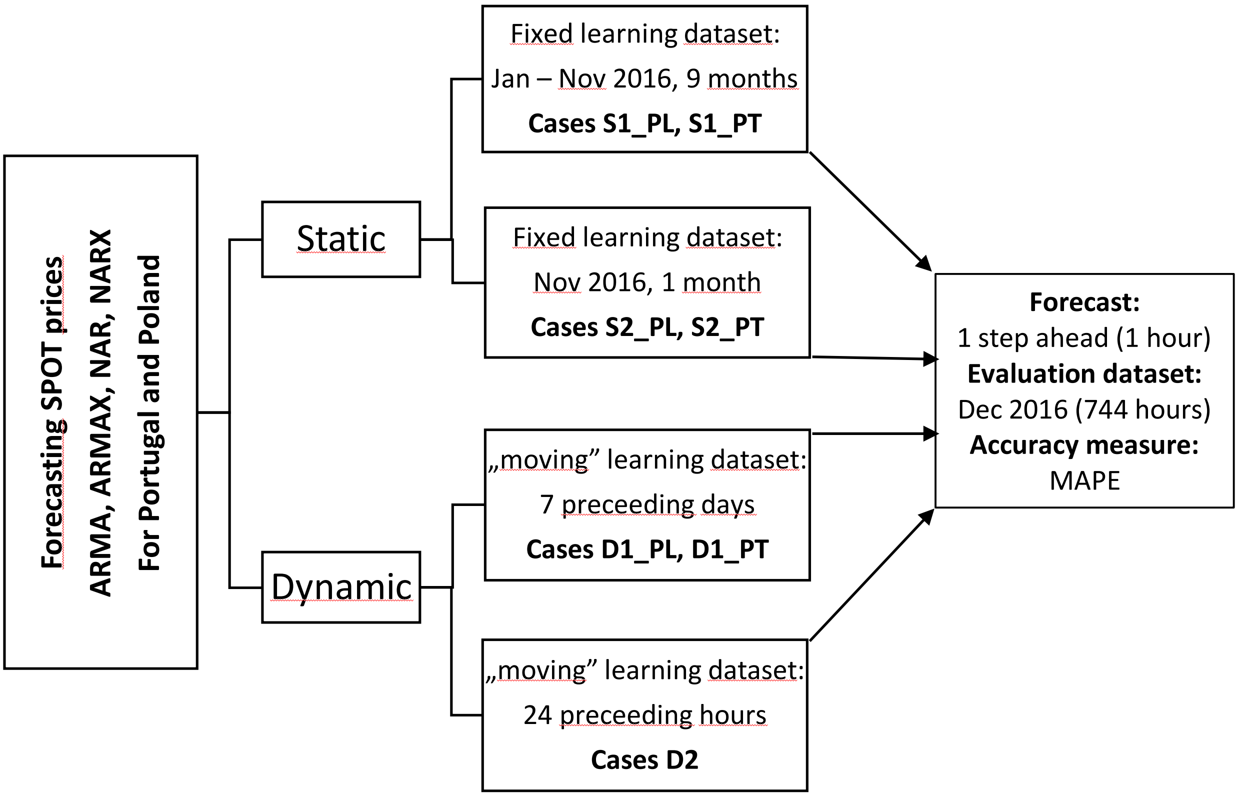


Figure 21 Schematic representation of the SPOT prices prediction cases

Firstly, the prognoses will be created for the "static" cases iteratively, varying the number of lags used in the models. The results of the forecasts carried out in this way will be analysed for finding the model which will be featured by the lowest MAPE value of the prediction. Then, the number of lags characterising the static model with the lowest MAPE will be applied in the dynamic model. The same rule will apply for the number of hidden layers in the case of ANN models.

5.4. Forecasting results

In order to have a reference to the models constructed in this project, the persistence model has been simulated as well. Applying the formulae described in equation 16 to the validation dataset (December

2016), the persistence model has been constructed and evaluated by obtaining the value of MAPE, which took the value of **6.92%** in Poland and **4.50%** in Portugal.

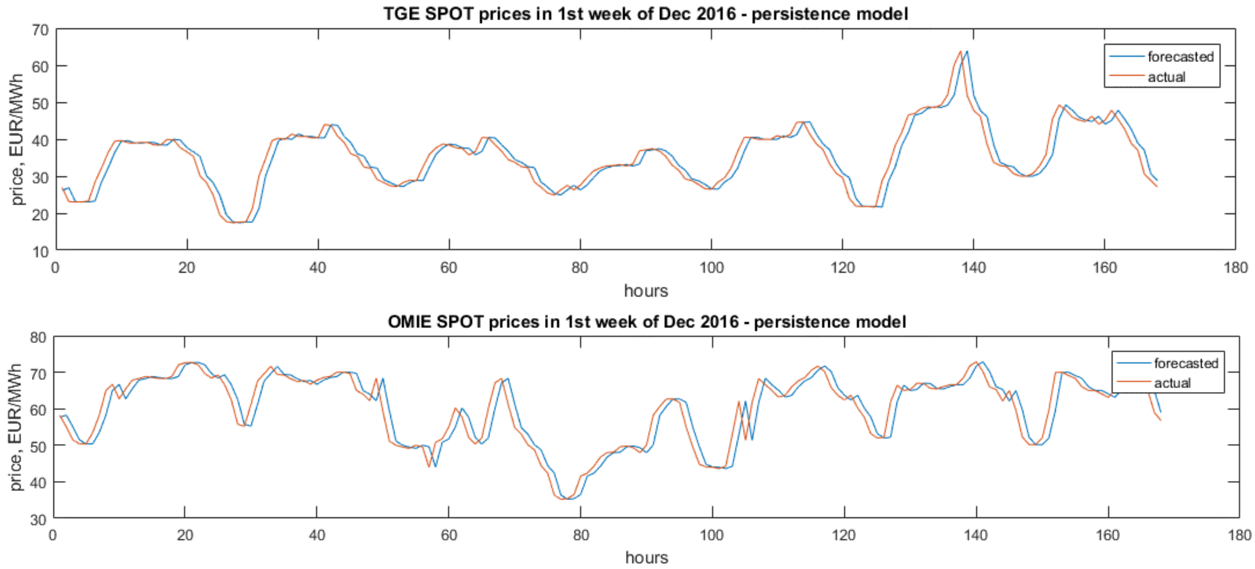


Figure 22 Persistence model SPOT price forecasting results for Poland and Portugal

At first glance, the achievement of the 4.5% MAPE in forecasting the Portuguese energy prices by means of naïve methods is considered as very effective.

Applying all the forecasting assumptions, there will be carried out 32 independent simulations of the models in order to predict the hour-ahead SPOT prices in December 2016 as the forecasting period. This number is derived from the number of models applied in this study, number of learning dataset approaches (see Figure 21) and the fact that the analysis is comparative for two countries – Poland and Portugal.

Since the ARMA model requires the stationarity of the subjected data, the SPOT prices time series have been integrated by subtraction of the price from hour *h-24* from price in hour, according to Eq. 23.

As the two main goals of this thesis were (i) examining the influence of wind energy forecast on SPOT prices and (ii) to what extent this relation takes place in the Polish and Portuguese markets. The forecasting result will be presented in order to reveal whether the models with external input variable (ARMAX and NARX) perform better in comparison with their counterparts without the wind generation forecast as an input variable (ARMA and NAR). Moreover, the analysis will include the comparison of the outcomes for the both countries, respectively.

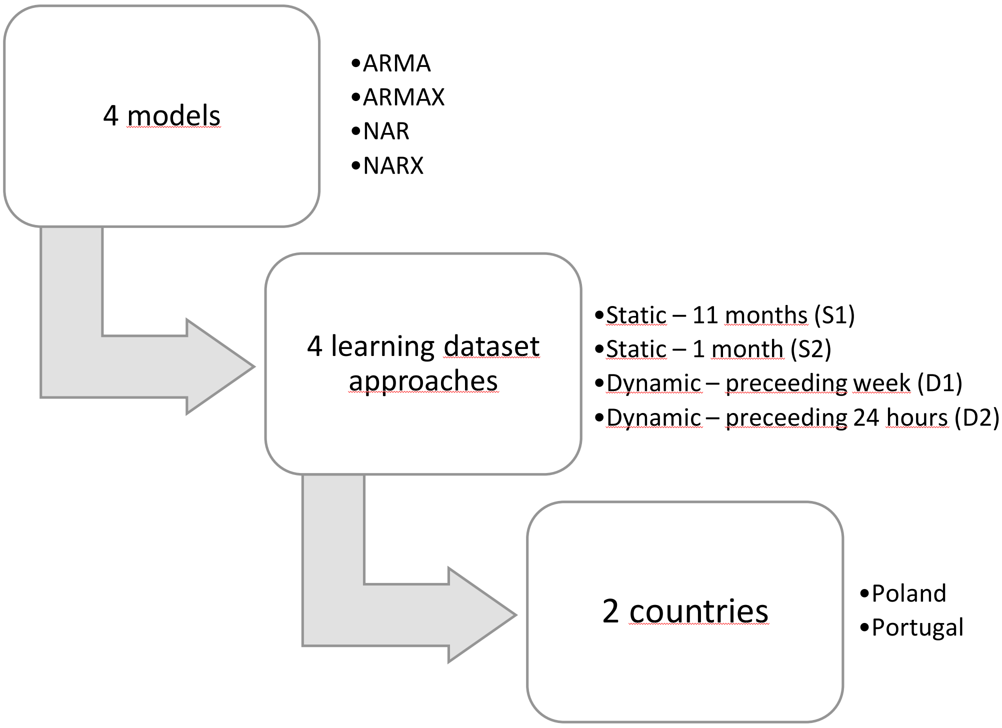


Figure 23 The breakdown of all forecasting simulations

Despite the testing dataset is 1 month (744 hours), the visualization of the forecasting results will be limited to showing the first week of the December (168 hours) in order to keep transparency of the graphs and enable the reader to compare visually the real data and forecasts outputs.

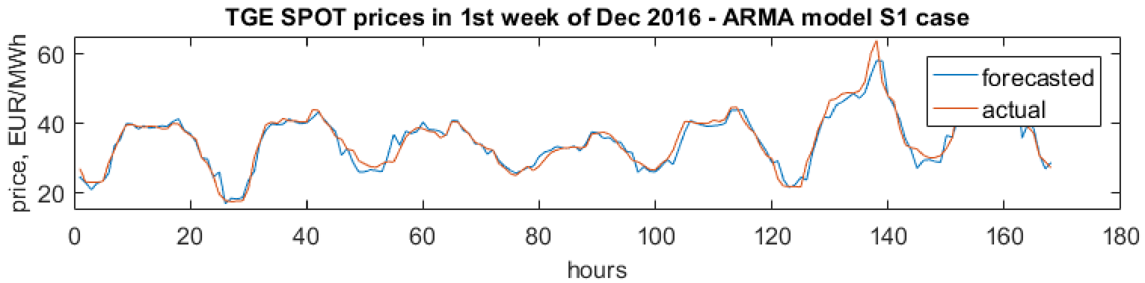
At the end of this chapter, the summarizing table will be included for cumulating of the results globally, what will allow to draw the overall conclusions and answer the main concerns of this thesis.

The Appendix A contains the exemplary MatLab codes which have been developed for the purpose of model estimation and forecasting, representative for the ARMA, ARMAX, NAR and NARX model.

The results are sequentially shown in the form of tables and adjacent plots, giving the information about the forecasting results of a given model for both countries.

Table 8 ARMA model S1 case SPOT price forecasting results for Poland and Portugal

ARMA model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	p	q		p	q	
S1	1	0	6.98%	5	4	3.66%



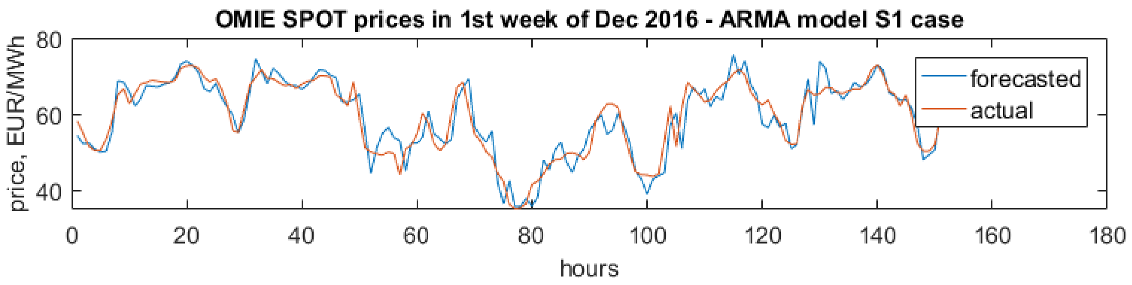
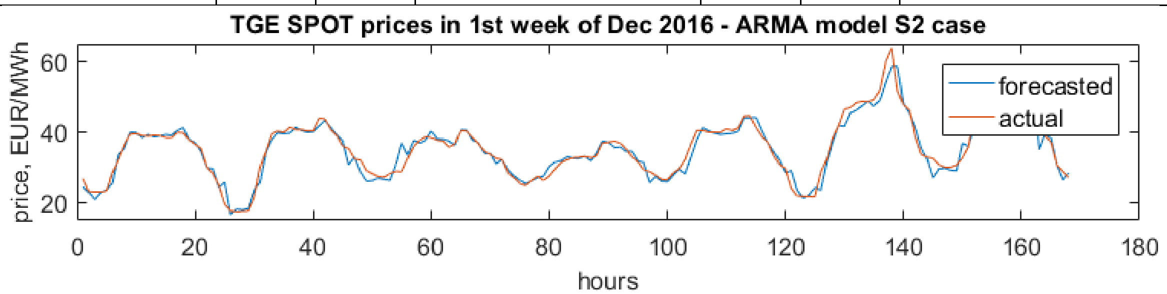


Table 9 ARMA model S2 case SPOT price forecasting results for Poland and Portugal

ARMA model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	p	q		p	q	
S2	1	0	6.90%	4	4	3.71%



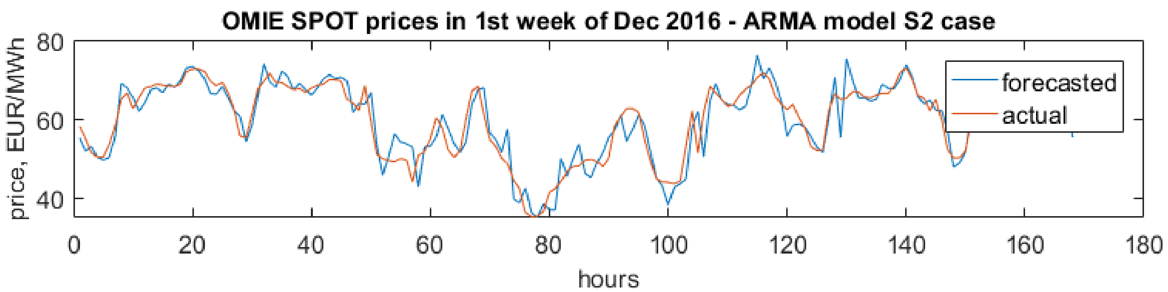
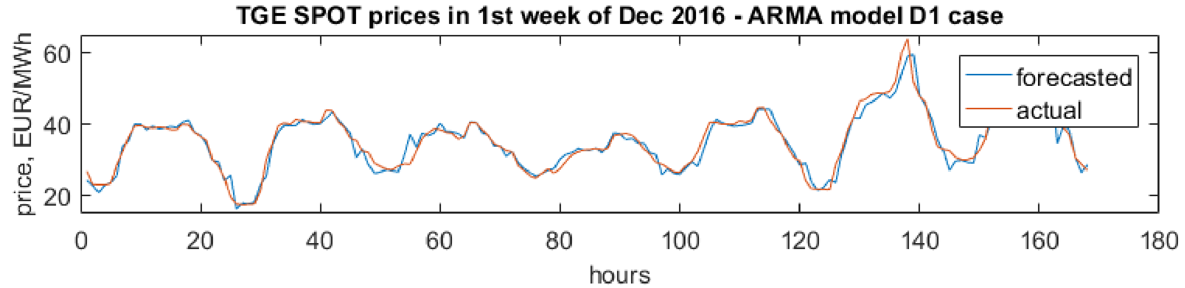


Table 10 ARMA model D1 case SPOT price forecasting results for Poland and Portugal

ARMA model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	p	q		p	q	
D1	1	2	7.04%	3	3	4.42%



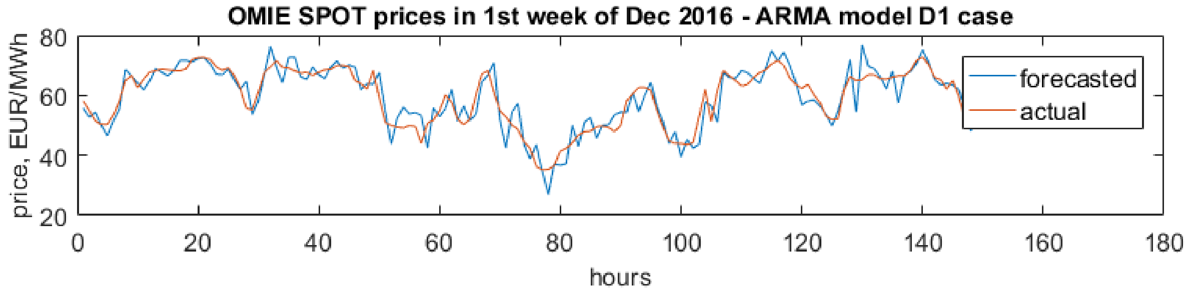
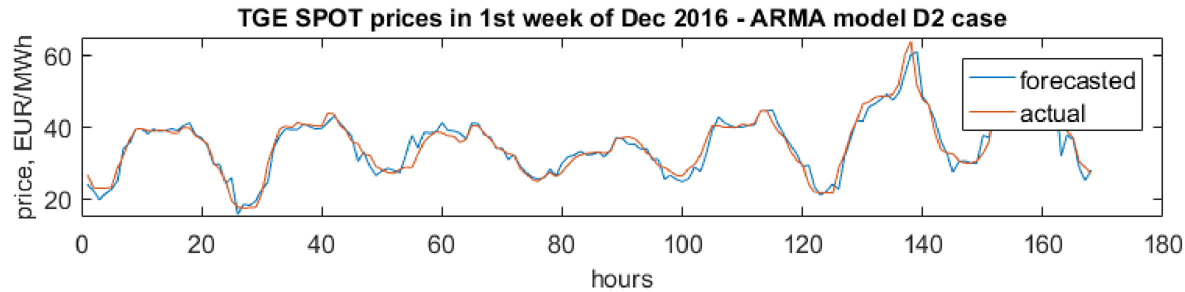
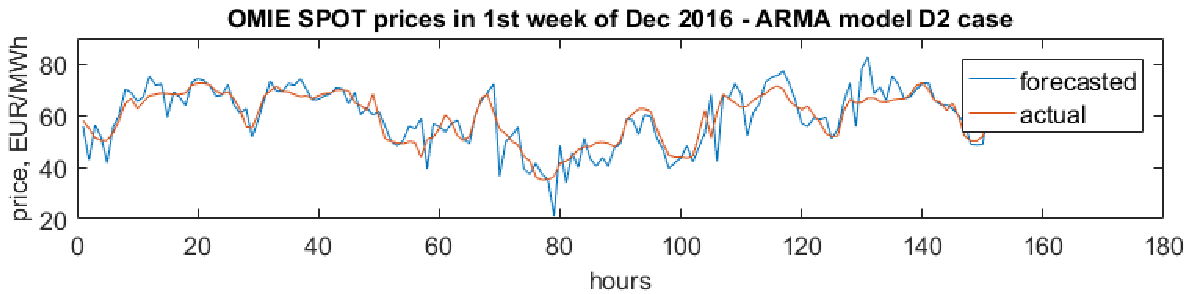


Table 11 ARMA model D2 case SPOT price forecasting results for Poland and Portugal

ARMA model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	p	q		p	q	
D2	1	0	8.23%	3	3	6.09%





Comments:

Comparing the paths of the SPOT prices time series in the representative first week of December 2016 for both countries, a much higher volatility within a single day can be observed for Portugal. Case of ARMA model for the cases S1 and S2 in Poland, the optimal model parameters were *p*=1 and *q*=0, what means that the Moving Average part of the model is unnecessary in these cases and the model becomes simply AR (AutoRegresive) one. As regards Portugal, the best models structures were basing on higher-order polynomials, varying from 3 to 5.

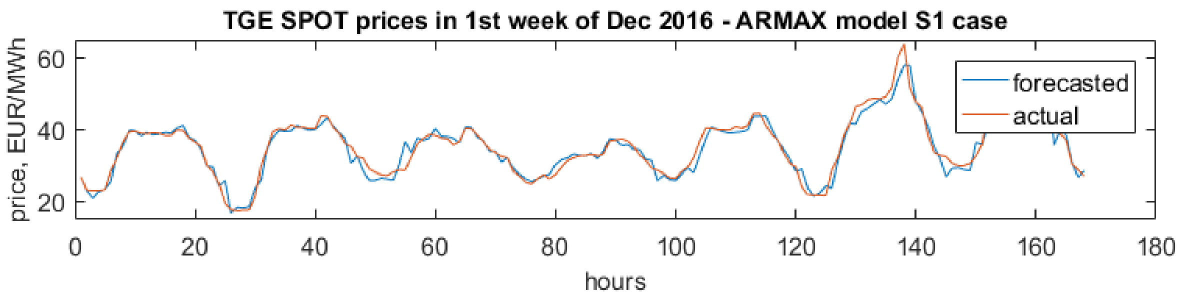
Despite the fact that the OMIE SPOT market prices show higher volatility, the time series ARMA model performed better on the Portuguese data, resulting in significantly lower MAPE values.

Analysing all the ARMA simulations it was observed that the best results of the forecasts have been obtained for the S1 case (11 months of past data for estimation), both for TGE (MAPE 6.98%) and OMIE(MAPE 3.66%). The cases when the learning set was updated every proceeding step of forecasting brought less accurate outputs.

Further, the simulation results have been shown for the ARMAX models.

Table 12 ARMAX model S1 case SPOT price forecasting results for Poland and Portugal

ARMAX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	na	nb	nc		na	nb	nc	
S1	1	0	0	6.97%	5	4	5	3.57%



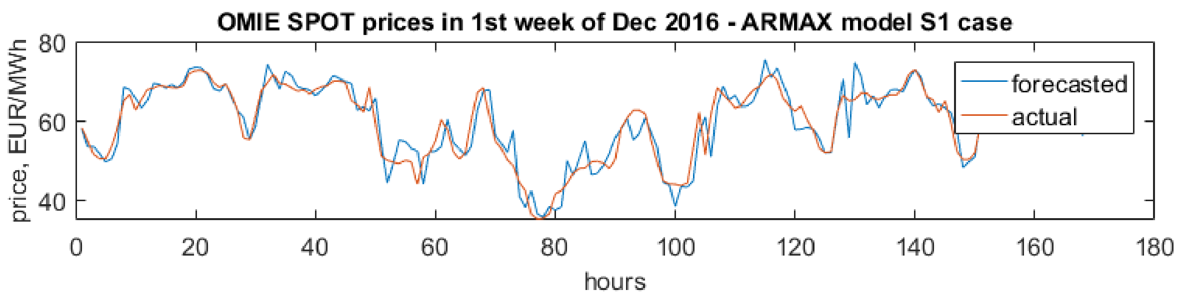
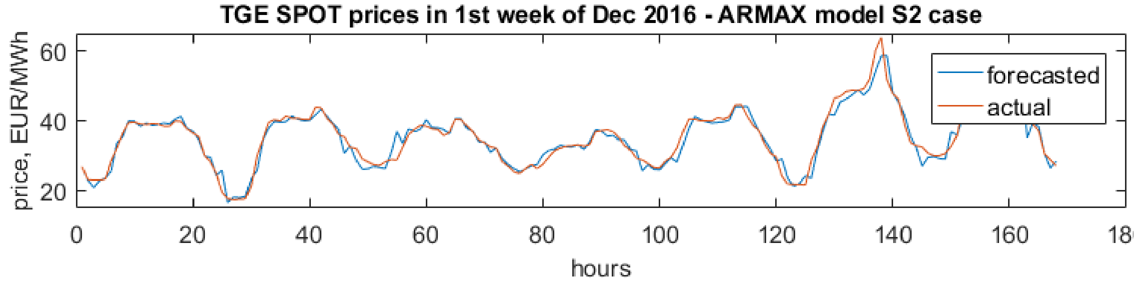


Table 13 ARMAX model S2 case SPOT price forecasting results for Poland and Portugal

ARMAX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	na	nb	nc		na	nb	nc	
S2	1	0	0	6.89%	5	2	5	3.57%



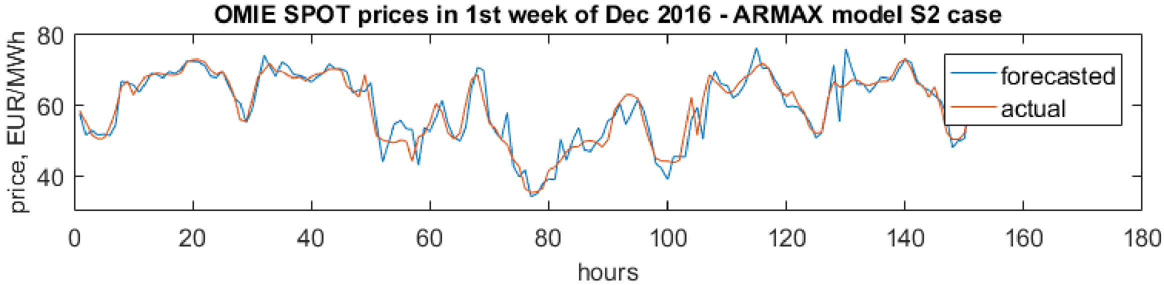
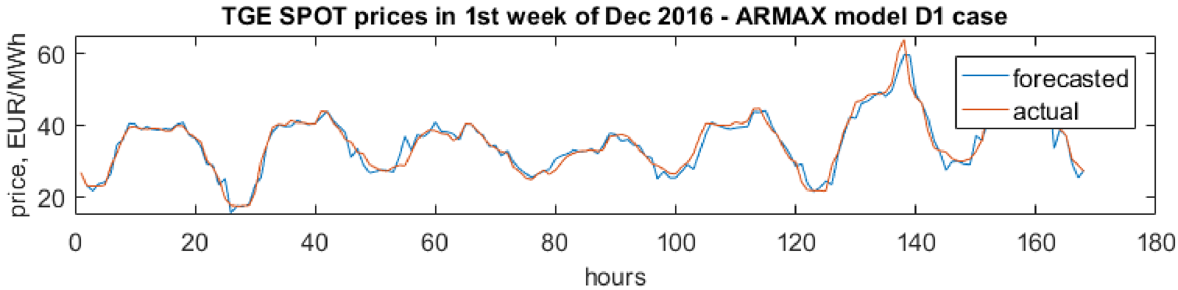


Table 14 ARMAX model D1 case SPOT price forecasting results for Poland and Portugal

ARMAX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	na	nb	nc		na	nb	nc	
D1	2	2	0	6.79%	4	2	4	3.64%



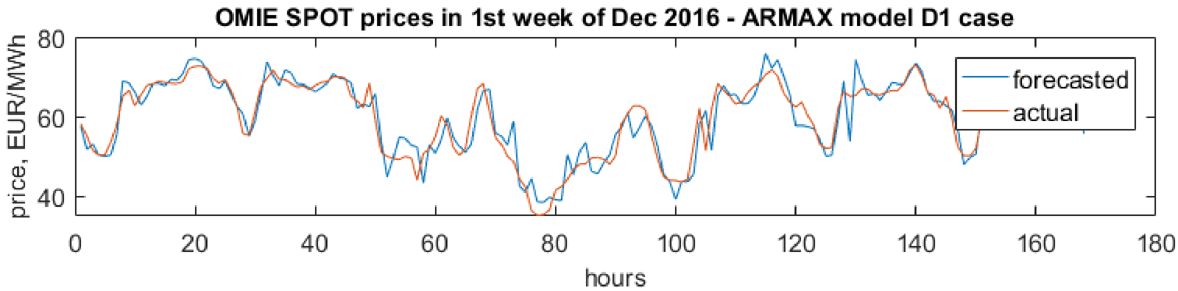
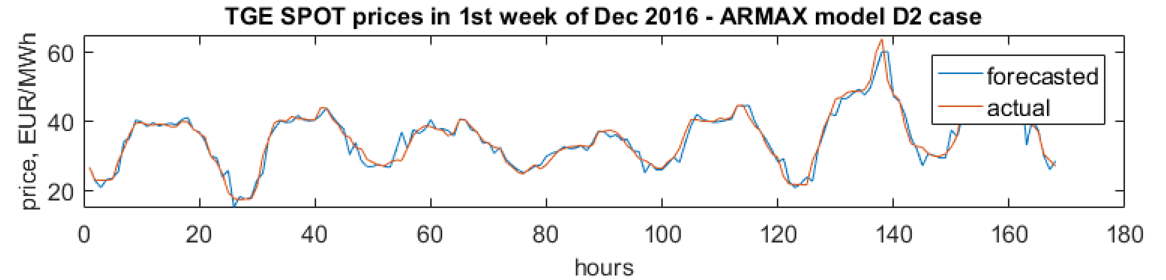
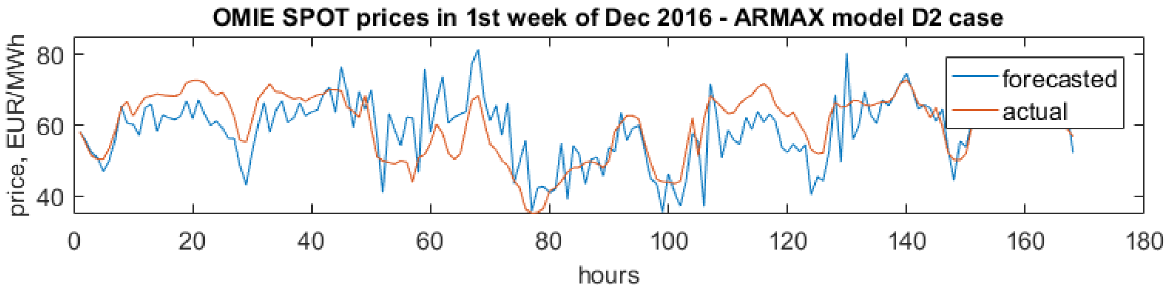


Table 15 ARMAX model D2 case SPOT price forecasting results for Poland and Portugal

ARMAX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	na	nb	nc		na	nb	nc	
D2	1	0	1	6.81%	3	5	3	7.77%





Comments:

Comparing the parameters of the ARMAX model between two countries (*na, nb, nc*) the models working on the Portuguese market data performs the best for the higher polynomial orders. Similarly, as in the application of the ARMA model, a smaller error can be noticed for the Portuguese prices forecasts.

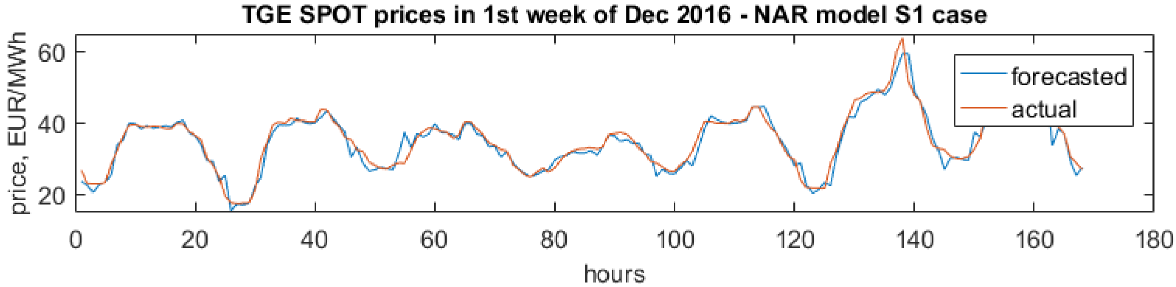
Except the case D1, in the ARMAX models for forecasting TGE SPOT price the best results have been obtained when the *nb* parameter was equal to 0, what means that the polynomial corresponding to the external input value was equal to zero, what finally leads to the conclusion that in these cases the wind forecast time series was not useful (see Eq. 27).

A significant deterioration of the forecast quality can be observed, comparing D1 and D2 cases of the OMIE price forecasting models, what is observable also on the attached graphs.

Moreover, in some instances of the Polish market prices ARMAX models the value of parameter *nc* is zero as well, what actually transforms the ARMAX model into AR model, without inclusion of the wind energy forecast input.

Table 16 NAR model S1 case SPOT price forecasting results for Poland and Portugal

NAR model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	Delays	Hidden layers		Delays	Hidden Layers	
S1	1	5	6.71%	5	1	3.74%



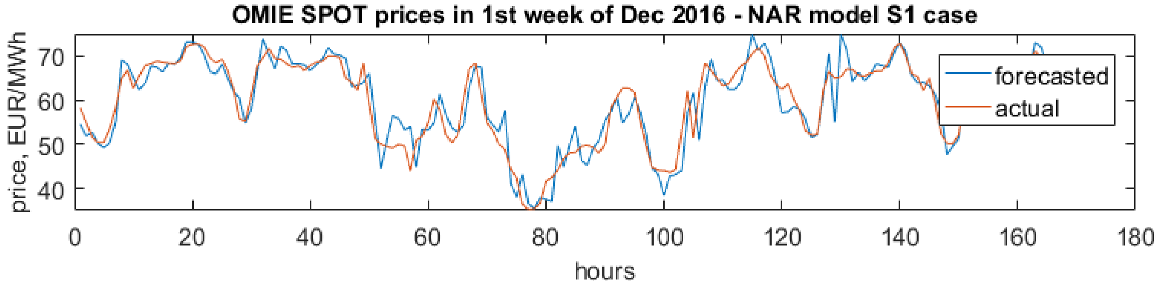
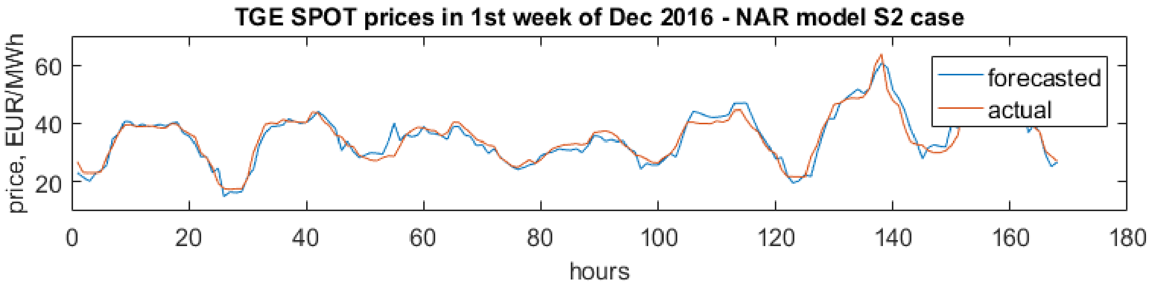


Table 17 NAR model S1 case SPOT price forecasting results for Poland and Portugal

NAR model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	Delays	Hidden layers		Delays	Hidden Layers	
S2	2	2	6.55%	4	5	3.71%



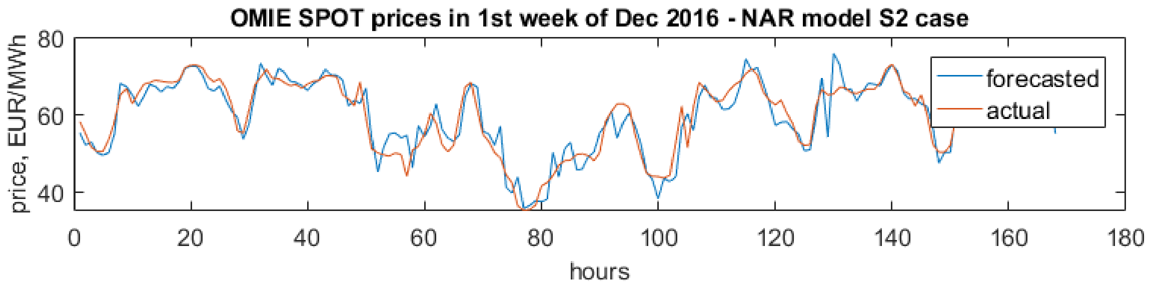
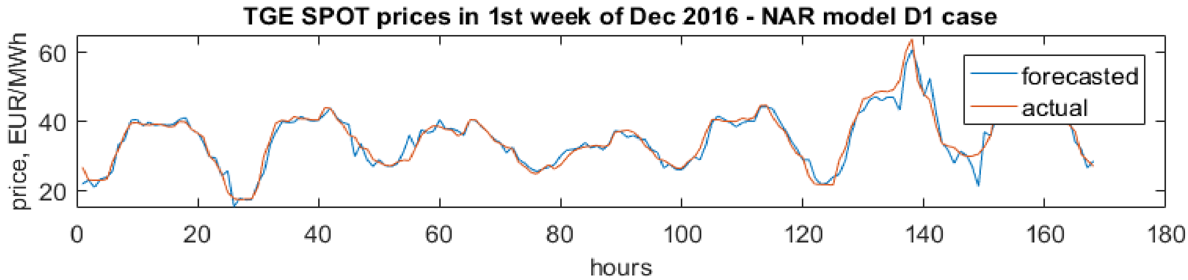


Table 18 NAR model D1 case SPOT price forecasting results for Poland and Portugal

NAR model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	Delays	Hidden layers		Delays	Hidden Layers	
D1	2	3	7.54%	1	1	3.88%



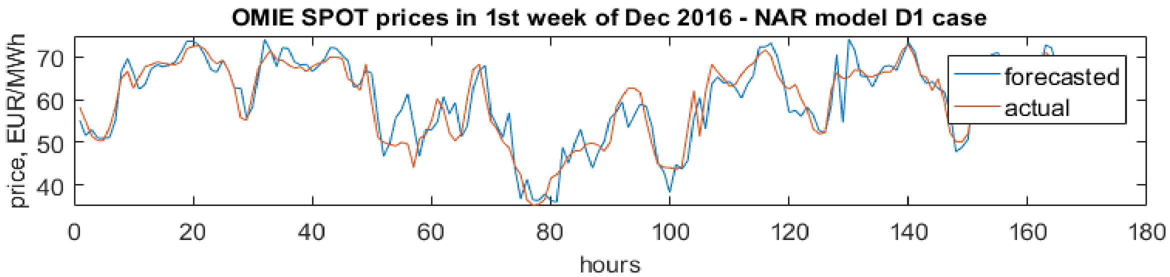
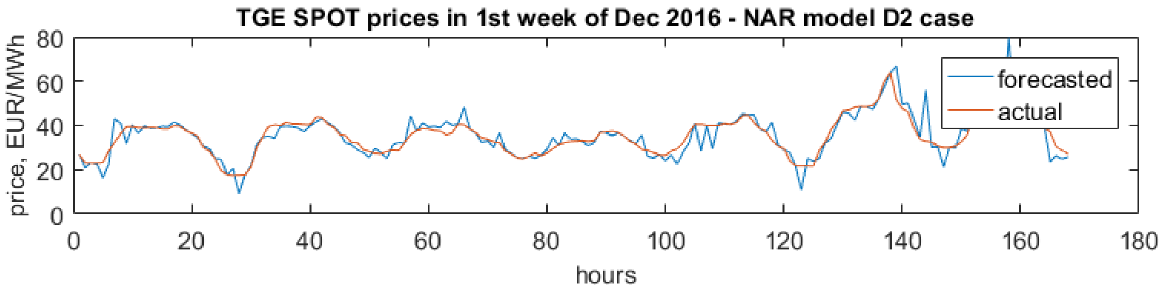
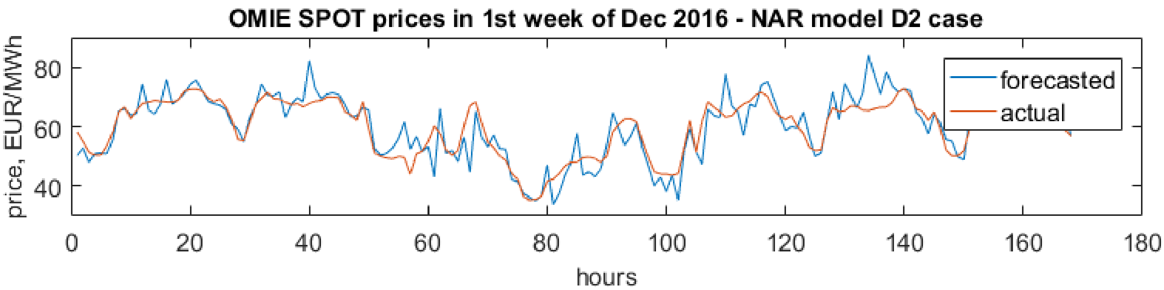


Table 19 NAR model D2 case SPOT price forecasting results for Poland and Portugal

NAR model						
CASE	POLAND			PORTUGAL		
	PARAMETERS		MAPE	PARAMETERS		MAPE
	Delays	Hidden layers		Delays	Hidden Layers	
D2	3	4	9.83%	3	4	4.58%





Comments:

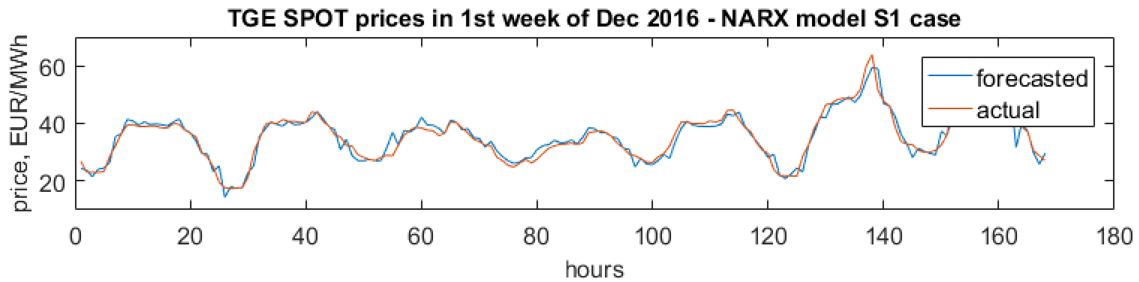
Analysis of the NAR model revealed the most certain forecasts have been developed for the S2 case, which was featured by 1 month (November 2016) estimation dataset. Similarly to the ARMA and ARMAX models, the dynamic approaches (D1 and D2 cases) have not improved the forecasts results. This is contrary to the expectations for the OMIE case especially, since there was observable an increasing trend of the prices in the entire year were increasing (see Figure 20). The every-hour model parameters update has not enhanced the models performance.

Following the remaining examined models, in the case of NAR model the more accurate forecast have been obtained for the Portuguese SPOT prices time series, unexpectedly because of the visibly higher volatility, comparing to the corresponding time series in Poland (see Figure 20).

Analysing the outputs of the NAR model carried out for Polish dataset, a general conclusion is that the ARMAX and ARMA model reveal better accuracy overall. In the case of Portuguese market, this advantage is not observed in all the examined cases.

Table 20 NARX model S1 case SPOT price forecasting results for Poland and Portugal

NARX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	delays	Ext. Input delays	hidden layers		delays	Ext. Input delays	hidden layers	
S1	3	5	2	6.74%	5	2	5	3.63%



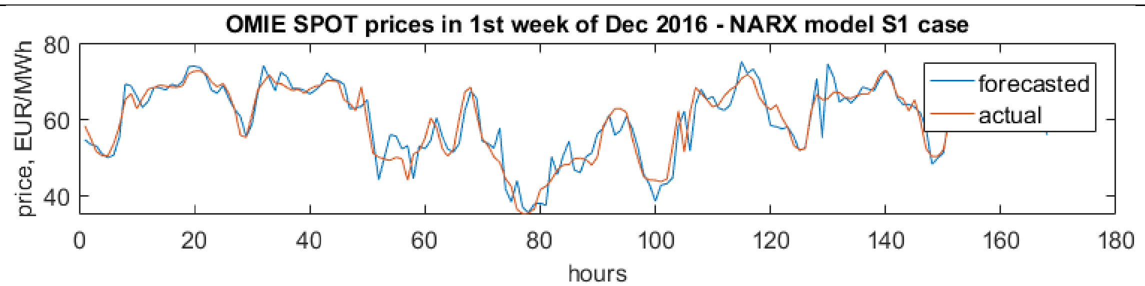
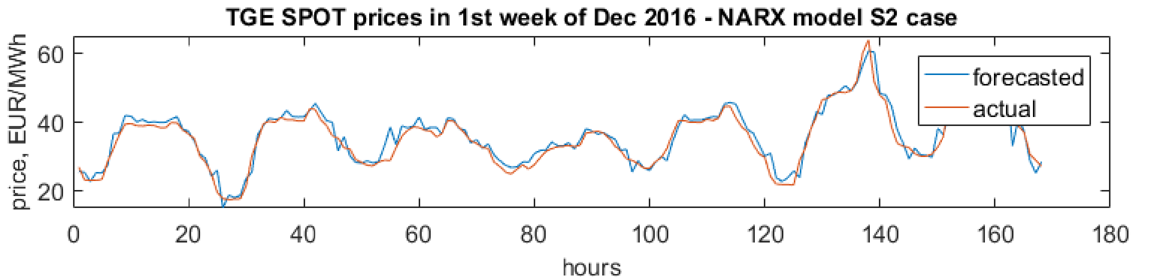


Table 21 NARX model S2 case SPOT price forecasting results for Poland and Portugal

NARX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	delays	Ext. Input delays	hidden layers		delays	Ext. Input delays	hidden layers	
S2	5	5	2	6.63%	4	5	3	3.64%



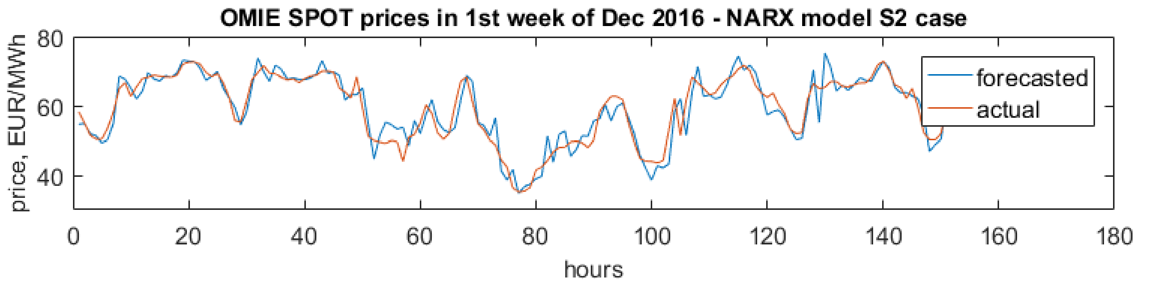
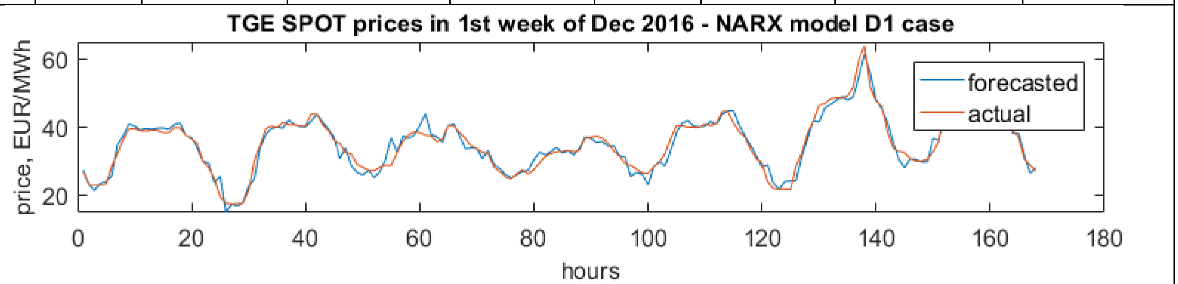


Table 22 NARX model D1 case SPOT price forecasting results for Poland and Portugal

NARX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	delays	Ext. Input delays	hidden layers		delays	Ext. Input delays	hidden layers	
D1	2	1	1	8.12%	5	5	5	3.67%



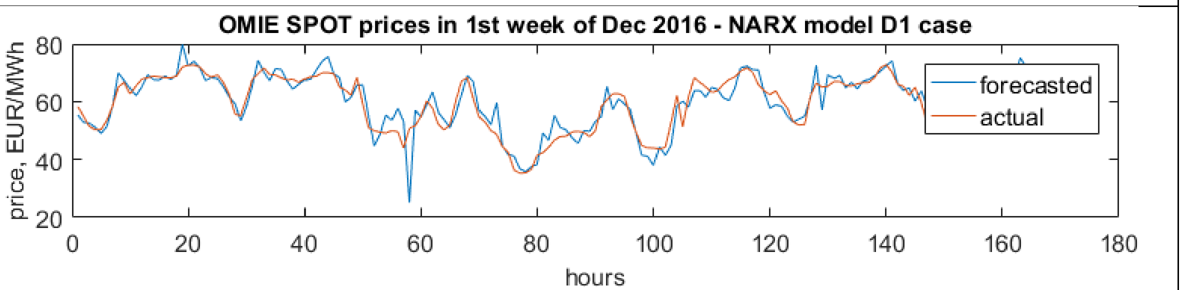
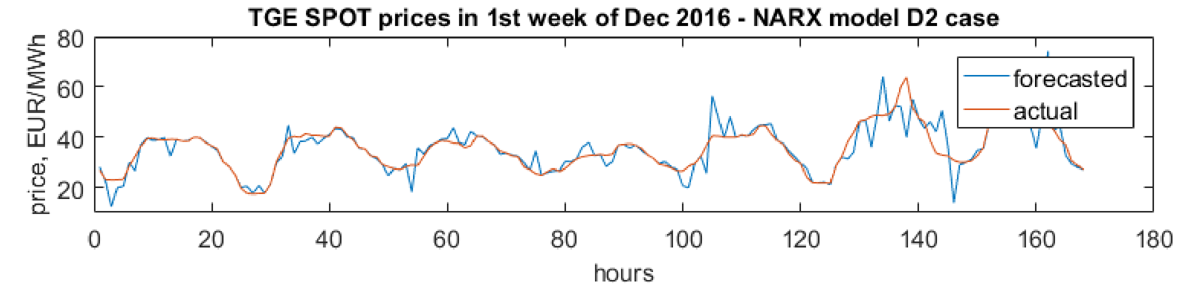
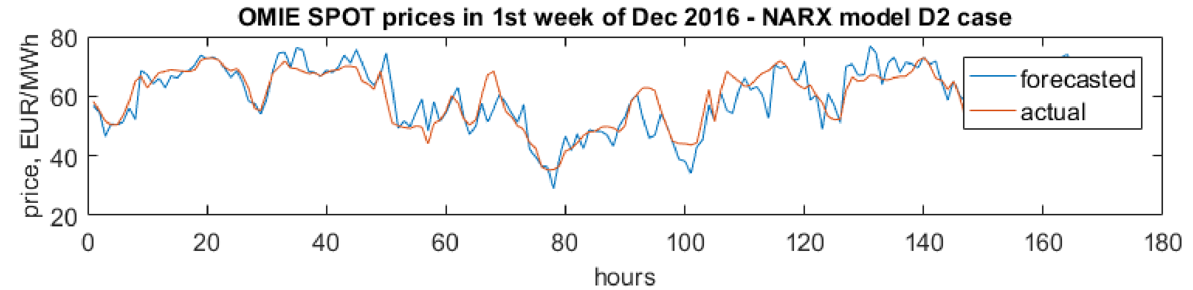


Table 23 NARX model D2 case SPOT price forecasting results for Poland and Portugal

NARX model								
CASE	POLAND				PORTUGAL			
	PARAMETERS			MAPE	PARAMETERS			MAPE
	delays	Ext. Input delays	hidden layers		delays	Ext. Input delays	hidden layers	
D2	4	4	4	8.78%	5	4	1	4.20%





Comments:

Comparison of the forecasting results among the cases set, a conclusion has been derived that the worst accuracy reflected in the highest MAPE value has been observed for the D2 case, which was using only 24 past records of the price for the model estimation. Forecasting process has been accomplished most successfully with use of wider range of data (S1 and S2 cases), for both Portugal and Poland.

Tracking the curves representing the forecasting outcomes versus actual data it can be noted that the models in some instances predict significantly excessive peaks, what certainly influences the worsened overall MAPE value in the horizon of whole December 2016.

For the purpose of summarizing the forecasting results and direct comparison of particular models with and without the “X” extension as wind generation forecast, the cumulative tables have been shown for all the analysed models and subsequent cases altogether.

Table 24 Comparison of ARMA and ARMAX SPOT prices forecasting models

POLAND										
CASE	ARMA				ARMAX					**SU_{ARMA}-SU_{ARMAX}**
	PARAMETERS		MAPE	SU_{ARMA}	PARAMETERS			MAPE	SU_{ARMAX}	
	p	q		[EUR]	na	nb	nc		[EUR]	[EUR]
S1	1	0	6.98%	3 448 565	1	0	0	6.97%	3 443 624	-
S2	1	0	6.90%	3 409 040	1	0	0	6.89%	3 404 099	-
D1	1	2	7.04%	3 478 209	2	2	0	6.79%	3 354 693	123 516
D2	1	0	8.23%	4 066 145	1	0	1	6.81%	3 364 574	-
PORTUGAL										
CASE	ARMA				ARMAX					**SU_{ARMA}-SU_{ARMAX}**
	PARAMETERS		MAPE	SU_{ARMA}	PARAMETERS			MAPE	SU_{ARMAX}	
	p	p		[EUR]	na	nb	nc		[EUR]	[EUR]
S1	5	4	3.66%	2 262 710	5	4	5	3.57%	2 207 070	55 640
S2	4	4	3.71%	2 293 622	5	2	5	3.57%	2 207 070	86 552
D1	3	3	4.24%	2 621 282	4	2	4	3.64%	2 250 346	370 936
D2	3	3	6.09%	3 765 002	3	5	3	7.77%	4 803 623	-

Table 25 Comparison of NAR and NARX SPOT prices forecasting models

POLAND										
CASE	NAR				NARX					**SU_{NAR}-SU_{NARX}**
	PARAMETERS		MAPE	SU_{NAR}	PARAMETERS			MAPE	SU_{NARX}	
	delays	hidden layers		[EUR]	delays	ex. Input delays	hidden layers		[EUR]	[EUR]
S1	1	5	6.71%	3 315 168	3	5	2	6.74%	3 329 990	-
S2	2	2	6.55%	3 236 118	5	5	2	6.63%	3 275 643	-
D1	2	3	7.54%	3 725 241	2	1	1	8.12%	4 011 798	-
D2	3	4	9.83%	4 856 647	4	4	4	8.78%	4 337 880	518 767
PORTUGAL										
CASE	NAR				NARX					**SU_{NAR}-SU_{NARX}**
	PARAMETERS		MAPE	SU_{NAR}	PARAMETERS			MAPE	SU_{NARX}	
	delays	hidden layers		[EUR]	delays	ex. Input delays	hidden layers		[EUR]	[EUR]
S1	5	1	3.74%	2 312 169	5	2	5	3.63%	2 244 164	68 005
S2	4	5	3.71%	2 293 622	4	5	3	3.64%	2 250 346	43 276
D1	1	1	3.88%	2 398 720	5	5	5	3.67%	2 268 893	129 828
D2	3	4	4.58%	2 831 479	5	4	1	4.20%	2 596 553	234 926

Comments:

In the Table 24 and Table 25, all the simulation results have been shown. The rows highlighted by light-blue colour indicate a situation, when the model with external input variable (wind generation forecast) performed better than the same model but basing only on the past values of SPOT prices. Despite the better forecast accuracy of the ARMAX model than the ARMA model in the S1, S2 and D2 cases in Poland, the corresponding results have not been highlighted – it should be pointed that in these cases the best performing ARMAX model structure was with *nb* polynomial order equal to 0, what means that the ARMAX model does not include the external variable at all and actually becomes ARMA model. Therefore, the improvement of these predictions cannot be associated with addition of the wind power forecast information. Additionally, in the right-adjacent column the difference between the Sales Uncertainty of the models without and with inclusion of the wind generation forecast has been calculated – as it can be observed, the sales uncertainty decrease varied between 55 000 EUR to around 235 000 EUR in the case of models based on Portuguese data.

Recalling the reference forecasts of the persistence model (6.92% MAPE in Poland and 4.50% in Portugal) it can be noticed that in the case of Poland the obtained models rarely perform better that the persistence one, while in Portugal the MAPE obtained by time series models is often below 4%.

Comparing country-to-country, it can be observed that the wind generation forecast is valuable for predicting the SPOT prices in Portugal, because in 7 out of 8 cases the "X" model returned more accurate model than its counterpart without information of wind generation forecast (ARMA vs ARMAX and NAR vs NARX). In the case of predicting the TGE SPOT market prices, only in 2 instances out of 8 the addition of wind production forecasts revealed to improve the model output. The cumulated simulation results show that the wind generation forecasts have an influence on the SPOT market prices in Portugal, which can be implemented in the mathematical models. In the case of Polish market, this statement cannot be made. As a source of this difference the author considers the significant divergence in the RES (and in consequence wind) generation share in the total electricity production. In the case of Portugal, the relative amount of the energy injected to the system from windmills is much higher, thus, may have a greater impact on the market behaviour.

As a general tendency, the OMIE SPOT market prices occurred to be more predictable than the TGE time series, what is expressed by one-sided difference of the MAPE measure in all the cases in favour of the Portuguese case.

Moreover, when comparing the results obtained from the four models for the individual learning dataset approaches (S1, S2, D1 and D2), a global observation is that the models of "static" character, it is being estimated once on a fixed dataset, resulted in obtaining lower errors. As the reason, the author assumes the instantaneous peaks of the prices occurring within a week or a day, what may remarkably influence the estimation and in consequence burden the model's prediction accuracy. In the situation when the time range of the data was wider (1 month, 11 months), this impact was affection was mitigated.

At this point it has to be highlighted that the above results obtained on the way of applying time series models, sometimes requiring several hours to conduct the simulation, in some instances performed worse than the persistence model (described in section 3.2), which for the TGE prices resulted in MAPE equal to 6.92% and for OMIE prices – 4.50%. The most accurate forecast for the TGE prices have been achieved by NAR model S2 case (6.55%), while for the OMIE prices by ARMAX model S1 case (3.55%).

6. Conclusions

On the way of literature study for the purposes of this thesis, a wide range of models for prediction of the wind energy has been found, often leading to obtain promising results. Most of these models were dedicated for forecasting the production of the wind energy by individual windfarms, or were aggregated for a region, where the windmills were in dense allocation. Despite the fact that the performance of forecasts for exemplary windfarms reveals satisfactory uncertainty, the aggregation of these predictions to the global (overall national level) scale results in significant forecasting errors, based on calculations made by use of ENTSOE platform. The remarkable discrepancies have been observed for both analysed countries. Analysis of the wind forecasts error distribution revealed that there exists a preference in underestimation of the wind generation forecast, i.e. the forecast more often predicted lower values than the actual ones. Additional analysis (Theil Divergence) revealed that the most of the wind generation forecasts uncertainty is influenced by inappropriate detection of switching the direction of actual trend.

Calculation of the financial losses led to the conclusion that the uncertainty of the wind energy production planning/forecasting lead to remarkable potential income losses for the wind energy producers, reaching millions of EUR in global(national) scale. However, a distinction has to be emphasised when comparing the results for Poland and Portugal – despite the fact that in overall, the wind generation forecast deviations resulted in financial losses in both countries, in Poland, in over half of the instances (examined hours of year 2016), the spread between the BM price and SPOT price resulted in additional income, compared to the situation where the perfectly planned production was sold only in the SPOT market. In other words, there exists a space for market speculation in Poland. In Portugal, this opportunity is limited – the beneficial spread was observed sporadically. The difference between Poland and Portugal comes from different BM pricing; in Poland, there is a single balancing price, the same for surplus/deficiency of produced energy, while on the other hand, in Portugal there are two separate BM prices, depending whether the producer exceeded/not completed the production plan.

Finally, the series of forecasts of SPOT market prices in Poland and Portugal using four different models allowed to conclude that the forecasted wind power injection to the system influences the day-ahead prices in Portugal in way possible to be modelled, what has been proven by improved accuracy of the prediction errors, what translated into significant reduction of the wind energy sale uncertainty, reaching in the extreme case around 235 000 EUR in Portugal for the period of December 2016. On the other hand, the same cannot be stated for Poland, where the models with inclusion of the wind generation forecasts have not revealed improvement. Contrary to the initial expectations of the author, the advantage of Neural Network models over polynomial-based models is not inequivocal.

In the Appendixes attached in the end of this document, the forecasting results within selected ranges of iteration have been shown, together with exemplary Matlab codes written for the purpose of this study.

Future Work

The presented study, based on basic models implemented in the accessible tools of Matlab software shown a potential of the wind generation forecast as a variable for predicting the electricity market prices, especially in the case of Portugal. As the expansion of the following project, the development of the hereby presented models by means of peak-detecting procedures and inclusion of more external key-variables (for example total load in the system) is considered, what could further improve the SPOT price forecasts.

The comparative analysis carried out within this thesis revealed a potential of commercial application – for example, in Poland, one can find offers providing IT solutions for performing predictions in the Iberian market. The analysis focused on detection of main differences among two individual market systems may be desirable from the perspective of comparison of commercial IT forecasting tools.

In the present thesis, the forecasts have been made in one step ahead prediction manner. Further work on this topic could include the influence of extended forecasting horizon on the accuracy of the obtained models (2, 3 hours ahead).

To make the results comparable between Poland and Portugal, the Polish prices had to be recalculated by the PLN/EUR ratio, which is a subject of the market dynamics – therefore, it was suggested to the author that the analysis of the influence of the PLN/EUR currency on the obtained SPOT market prices forecasting should be verified.

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Appendix A – SPOT prices forecasting results for selected iteration ranges

ARMA model S1 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
p	q		p	q	
1	0	6.98%	3	3	3.74%
1	1	7.04%	3	4	5.56%
1	2	7.08%	3	5	3.71%
1	3	7.15%	4	0	3.75%
1	4	7.15%	4	1	3.75%
3	0	7.06%	5	3	3.71%
3	1	7.10%	5	4	3.66%
3	2	7.15%	5	5	3.67%

ARMA model S2 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
p	q		p	q	
1	0	6.90%	3	3	3.73%
1	1	7.03%	3	4	3.76%
1	2	7.13%	3	5	3.76%
2	4	7.18%	5	1	3.73%
2	5	7.41%	5	2	3.77%
3	0	7.16%	5	3	3.75%
3	1	7.19%	5	4	3.74%
3	2	7.20%	5	5	5.35%

ARMA model D1 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
p	q		p	q	
0	1	15.13%	3	3	4.24%
0	2	10.77%	3	4	4.31%
1	0	7.34%	3	5	5.85%
1	1	7.53%	4	3	4.36%
1	2	7.04%	4	4	5.99%
2	0	7.37%	4	5	6.30%
2	1	9.60%	5	3	5.72%
2	2	8.10%	5	4	6.04%

ARMA model D2 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
p	q		p	q	
0	1	15.57%	3	3	6.09%
0	2	12.78%	3	4	6.60%
1	0	8.23%	3	5	6.30%
1	1	9.21%	4	3	6.36%
1	2	9.39%	4	4	6.29%
2	0	9.11%	4	5	6.60%
2	1	10.14%	5	3	6.14%
2	2	11.96%	5	4	6.51%

ARMAX model S1 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
na	nb	nc		na	nb	nc	
0	5	4	8.10%	5	4	2	3.67%
0	5	5	7.82%	5	4	3	3.61%
1	0	0	6.97%	5	4	4	3.60%
1	0	1	7.03%	5	4	5	3.57%
1	0	2	7.07%	5	5	0	3.68%
1	0	3	7.13%	5	5	1	3.68%
1	0	4	7.12%	5	5	2	3.66%
1	0	5	7.12%	5	5	3	3.67%

ARMAX model S2 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
na	nb	nc		na	nb	nc	
0	5	4	8.15%	5	2	1	3.68%
0	5	5	7.85%	5	2	2	3.68%
1	0	0	6.89%	5	2	3	3.60%
1	0	1	7.00%	5	2	4	3.72%
1	0	2	7.09%	5	2	5	3.57%
1	0	3	7.14%	5	3	0	3.66%
1	0	4	7.14%	5	3	1	3.68%
1	0	5	7.17%	5	3	2	3.68%

ARMAX model D1 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
na	nb	nc		na	nb	nc	
2	0	1	6.85%	3	5	5	6.54%
2	0	2	6.91%	4	2	2	3.76%
2	1	0	6.84%	4	2	3	3.81%
2	1	1	6.91%	4	2	4	3.64%
2	1	2	6.94%	4	2	5	3.70%
2	2	0	6.79%	4	3	2	3.80%
2	2	1	6.84%	4	3	3	3.67%
2	2	2	6.83%	4	3	4	5.47%

ARMAX model D2 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
na	nb	nc		na	nb	nc	
0	2	1	21.25%	3	3	3	140.26%
0	2	2	13.81%	3	3	4	96.03%
1	0	0	6.85%	3	3	5	131.19%
1	0	1	6.81%	3	4	3	92.02%
1	0	2	20.10%	3	4	4	47.73%
1	1	0	7.60%	3	4	5	47.13%
1	1	1	7.44%	3	5	3	7.77%
1	1	2	33.79%	3	5	4	169.53%

NAR model S1 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
Delays	Hidden layers		Delays	Hidden layers	
1	1	6.82%	4	3	3.74%
1	2	6.83%	4	4	3.75%
1	3	6.96%	4	5	3.77%
1	4	6.99%	5	1	3.74%
1	5	6.71%	5	2	3.74%
2	1	7.09%	5	3	3.77%
2	2	6.86%	5	4	3.74%
2	3	10.90%	5	5	3.77%

NAR model S2 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
Delays	Hidden layers		Delays	Hidden layers	
1	5	7.22%	4	1	3.73%
2	1	6.66%	4	2	3.74%
2	2	6.55%	4	3	3.75%
2	3	6.69%	4	4	3.74%
2	4	6.76%	4	5	3.71%
2	5	6.86%	5	1	3.95%
3	1	7.00%	5	2	3.71%
3	2	6.93%	5	3	3.79%

NAR model D1 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
Delays	Hidden layers		Delays	Hidden layers	
2	1	8.57%	1	1	3.88%
2	2	9.06%	1	2	4.21%
2	3	7.54%	1	3	4.04%
2	4	8.13%	1	4	4.05%
2	5	8.45%	1	5	4.13%
3	1	8.42%	2	1	4.04%
3	2	8.77%	2	2	3.93%
3	3	8.67%	2	3	3.94%

NAR model D2 case					
POLAND			PORTUGAL		
PARAMETERS		MAPE	PARAMETERS		MAPE
Delays	Hidden layers		Delays	Hidden layers	
3	1	11.58%	2	4	5.52%
3	2	12.38%	2	5	4.89%
3	3	10.86%	3	1	5.38%
3	4	9.83%	3	2	5.63%
3	5	11.43%	3	3	4.77%
4	1	11.87%	3	4	4.58%
4	2	13.24%	3	5	5.23%
4	3	12.09%	4	1	5.52%

NARX model S1 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
delays	Ex. Input delays	Hidden layers		Delays	Ex. Input delays	Hidden layers	
3	1	2	6.88%	5	3	4	3.64%
3	2	2	6.91%	5	4	4	3.67%
3	3	2	7.10%	5	5	4	3.70%
3	4	2	7.01%	5	1	5	3.64%
3	5	2	6.74%	5	2	5	3.63%
3	1	3	6.82%	5	3	5	3.69%
3	2	3	7.76%	5	4	5	3.64%
3	3	3	7.14%	5	5	5	3.67%

NARX model S2 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
delays	Ex. Input delays	Hidden layers		Delays	Ex. Input delays	Hidden layers	
5	2	2	16.50%	4	4	2	3.76%
5	3	2	7.45%	4	5	2	3.70%
5	4	2	9.89%	4	1	3	3.70%
5	5	2	6.63%	4	2	3	3.76%
5	1	3	7.65%	4	3	3	3.69%
5	2	3	6.81%	4	4	3	3.75%
5	3	3	7.40%	4	5	3	3.64%
5	4	3	7.71%	4	1	4	3.70%

NARX model D1 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
delays	Ex. Input delays	Hidden layers		Delays	Ex. Input delays	Hidden layers	
1	4	5	8.89%	5	3	4	3.93%
1	5	5	9.53%	5	4	4	3.96%
2	1	1	8.12%	5	5	4	3.88%
2	2	1	9.24%	5	1	5	3.94%
2	3	1	8.92%	5	2	5	3.85%
2	4	1	9.09%	5	3	5	3.94%
2	5	1	8.93%	5	4	5	3.70%
2	1	2	8.50%	5	5	5	3.67%

NARX model D2 case							
POLAND				PORTUGAL			
PARAMETERS			MAPE	PARAMETERS			MAPE
delays	Ex. Input delays	Hidden layers		Delays	Ex. Input delays	Hidden layers	
4	2	4	11.21%	5	2	1	5.56%
4	3	4	11.31%	5	3	1	5.09%
4	4	4	8.78%	5	4	1	4.20%
4	5	4	10.27%	5	5	1	5.10%
4	1	5	10.79%	5	1	2	5.89%
4	2	5	11.82%	5	2	2	5.03%
4	3	5	10.44%	5	3	2	4.77%
4	4	5	11.04%	5	4	2	4.79%

Appendix B – MatLab exemplary codes developed for the purpose of forecasting models.

Static ARMA model

```
W=zeros(744,1); %model results matrix pre-definition
MAPEs=zeros(25,3); %matrix with MAPE values
m=0; %auxiliary variable
likelihood=zeros(25,2); %matrix with values of likelihood function
h = waitbar(0,'please wait...');

for i=0:5 %order of AR model
   for j=0:5 %order of AM model

       model=arima(i,0,j);

[estmdl,EstParamCov,logL,info]=estimate(model,TGE_SPOT_PL_diff(1:8016));
       m=m+1;
       waitbar(m / 27)
       akaike(m,1)=i;
       akaike(m,2)=j;
       akaike(m,3)=logL;
       [aic,bic]=aicbic(logL,i+j+2,8016);
       akaike(m,4)=aic;
       akaike(m,5)=bic;
       likelihood(m,1)=logL;
       likelihood(m,2)=i+j+2;
          for u=1:744     %forecasting loop
              sample=TGE_SPOT_PL_diff((1+u): (8015+u));
              result=forecast(estmdl,1,'Y0',sample);
              W(u)=result;
          end

        FOR_TGE_SPOT_PL=TGE_SPOT_PL(7993:8736)+W;
        sum=0;

          for u=1:744 % calculation of total error
             err=abs(TGE_SPOT_PL(8016+u)-
             FOR_TGE_SPOT_PL(u))/TGE_SPOT_PL(8016+u);
             sum=sum+err;
         end

       MAPE=sum/744;
       MAPEs(m,1)=i;
       MAPEs(m,2)=j;
       MAPEs(m,3)=MAPE;

   end
end
close(h)

plot(FOR_TGE_SPOT_PL(1:168))
hold on
plot(TGE_SPOT_PL(8017:8184))
title('TGE SPOT prices in 1st week of Dec 2016 - ARMA model S1 case')
ylabel('price, EUR/MWh')
xlabel('hours')
legend('forecasted','actual')
filename = 'ARMA_S1_PL.xlsx';
xlswrite(filename,MAPEs);
```

Dynamic ARMAX model

```
W=iddata(744,[],1);
MAPEs=zeros(27,4);
m=0;
%transformation of data from double to idd type
totdat=iddata(TGE_SPOT_PL_diff,FOR_wind_gen_PL);
data=iddata(TGE_SPOT_PL_diff(1:8016),FOR_wind_gen_PL(1:8016),1);
val_data=iddata(TGE_SPOT_PL_diff(8017:end),FOR_wind_gen_PL(8017:end),1);

h = waitbar(0,'Please wait...');
AIC_BIC=zeros(125,5);% Akaike and Bayesian information matrix pre-
definition
for i=0:5 %order of A polynomial
  for j=0:5 %order of B polynomial
       for k=0:5 %order of C polynomial

            for u=1:744
                        data=iddata(TGE_SPOT_PL_diff(8015+u-
            168:8015+u),FOR_wind_gen_PL(8015+u-168:8015+u),1);

            val_data=iddata(TGE_SPOT_PL_diff(8017:end),FOR_wind_gen_PL(8017
            :end),1);

            model=armax(data,[i j k 0]); %model estimation functio
                   akaike=model.report.fit.AIC;
                   bayesi=model.report.fit.BIC;
                   AIC_BIC(m,4)=akaike;
                   AIC_BIC(m,5)=bayesi;
                   AIC_BIC(m,1)=i;
                   AIC_BIC(m,2)=j;
                   AIC_BIC(m,3)=k
            results=predict(model,val_data,1);
            res=results.y(:,1);%transform from idd data to double
            wynik=res(1:744)+TGE_SPOT_PL(7993:8736);
            end
      sum=0;
      m=m+1;
      waitbar(m/27)

        %calculation of the forecasting errors
        for u=1:744
        err=abs(TGE_SPOT_PL(8016+u)-wynik(u))/TGE_SPOT_PL(8016+u);
        sum=sum+err;
        end

       MAPE=sum/744;
       MAPEs(m,1)=i;
       MAPEs(m,2)=j;
       MAPEs(m,3)=k;
       MAPEs(m,4)=MAPE;
       end
  end
end
```

Static NAR model

```
%   adjust time series to model archotecture
T = tonndata(TGE_SPOT_PL_diff,false,false);
MAPEs=zeros(25,3);
h = waitbar(0,'PLease wait...');
m=0;
for i=1:5 %number of delays
    for j=1:5 %number of hidden layers

trainFcn = 'trainlm';  % Levenberg-Marquardt backpropagation.
feedbackDelays = 1:i;
hiddenLayerSize = j;
net = narnet(feedbackDelays,hiddenLayerSize,'open',trainFcn);
% Prepare the Data for Training and Simulation
 [x,xi,ai,t] = preparets(net,{},{},T);
% Setup Division of Data for Training, Validation, Testing
net.divideParam.trainRatio = 90/100;
net.divideParam.valRatio = 8/100;
net.divideParam.testRatio = 8/100; % 8% of total data is one month, Dec
% Train the Network
[net,tr] = train(net,x,t,xi,ai);
% Test the Network
y = net(x,xi,ai);
performance = perform(net,t,y);

wyn=transpose(cell2mat(y)); %transform from cell to double data

m=m+1;
waitbar(m/25);
raz=TGE_SPOT_PL(7993:8736);
dwa=wyn(end-743:end);
    FOR_TGE_SPOT_PL=raz+dwa;
       sum=0;
       %calculation of the error
       for u=1:744
       err=abs(TGE_SPOT_PL(8016+u)-FOR_TGE_SPOT_PL(u))/TGE_SPOT_PL(8016+u);
       sum=sum+err;
       end

      MAPE=sum/744;
      MAPEs(m,1)=i;
      MAPEs(m,2)=j;
      MAPEs(m,3)=MAPE;
    end
end

close(h)

plot(FOR_TGE_SPOT_PL(1:168))
hold on
plot(TGE_SPOT_PL(8017:8184))
title('TGE SPOT prices in 1st week of Dec 2016 - NAR model S1 case')
ylabel('price, EUR/MWh')
xlabel('hours')
legend('forecasted','actual')

filename = 'NAR_S1_PL.xlsx';
xlswrite(filename,MAPEs);
```

Dynamic NARX model

```
%   FOR_wind_gen_PL - input time series.
%   TGE_SPOT_PL_diff - feedback time series.
X = tonndata(FOR_wind_gen_PL,false,false);
T = tonndata(TGE_SPOT_PL_diff,false,false);
trainFcn = 'trainlm';  % Levenberg-Marquardt backpropagation.
MAPEs=zeros(125,4);
h = waitbar(0,'PTease wait...');
m=0;

for i=1:5 %number of delays
    for k=1:5%feedback delays
        for j=1:5 %number of hidden layers

            for u=1:744

            X = tonndata(FOR_wind_gen_PL(8014+u-168:8016+u),false,false);
            T = tonndata(TGE_SPOT_PL_diff(8014+u-168:8016+u),false,false);

% Create a Nonlinear Autoregressive Network with External Input
inputDelays = 1:k;
feedbackDelays = 1:i;
hiddenLayerSize = j;
net = narxnet(inputDelays,feedbackDelays,hiddenLayerSize,'open',trainFcn);
% Prepare the Data for Training and Simulation
 [x,xi,ai,t] = preparets(net,X,{},T);
% Setup Division of Data for Training, Validation, Testing
net.divideParam.trainRatio = 90/100;
net.divideParam.valRatio = 5/100;
net.divideParam.testRatio = 5/100;
% Train the Network
[net,tr] = train(net,x,t,xi,ai);
% Test the Network
y = net(x,xi,ai);
performance = perform(net,t,y);
wy=transpose(cell2mat(y)); %transform from cell to double type data
wyn(u,1)=wy(end,1);

            end

raz=TGE_SPOT_PL(7993:8736);
dwa=wyn(end-743:end);
FOR_TGE_SPOT_PL=raz+dwa;
       m=m+1;
       sum=0;
       waitbar(m/125)
       for u=1:744
       err=abs(TGE_SPOT_PL(8016+u)-FOR_TGE_SPOT_PL(u))/TGE_SPOT_PL(8016+u);
       sum=sum+err;
       end
                   MAPE=sum/744;
                   MAPEs(m,1)=i;
                   MAPEs(m,2)=j;
                   MAPEs(m,3)=k;
                   MAPEs(m,4)=MAPE;
    end
    end
end
```

Appendix C – Representative code for ARMA model parameters estimation

```
for i=0:6 %varying order of AR model
   for j=0:6 %varying order of AM model

      model=arima(i,0,j); %generating a model structure
      [estmdl,logL]=estimate(model,OMIE_SPOT_PT_diff(1:8016))
                          %estimating polynomial orders
      m=m+1; %iteration count -  auxiliary variable

      akaike(m,1)=i; %write p order into results matrix
      akaike(m,2)=j; %write p order into results matrix
      akaike(m,3)=logL; %logarithm of L likelihood function
      akaike(m,4)=aicbic(logL,i+j+2); %calculation of AIC
      akaike(m,5)=aicbic(logL,i+j+2,8016); %calculation of BIC

   end
end
```